$p=1, 2, 3, \cdots$, and put $G_p = |g_p|^2/R(g_p)$, $\tau_p = I(g_p)/R(g_p)$. Let W denote the open region exterior to the cut along the real axis from -1 to $-\infty$, and let $(1+z)^{1/2}$ be the branch of the square root which is 1 for $z=0$. The continued fraction $1/1 + G_1z/(1 - i\tau_1(1+z)^{1/2}) + (1 - G_1)G_2z/(1 - i\tau_2(1+z)^{1/2}) + (1 - G_2)G_3z/(1 - i\tau_3(1+z)^{1/2}) + \cdots$ converges uniformly over every bounded closed region in W. The class of functions $F(z)$ which are analytic and have positive real parts in W, and equal 1 for $z=0$, is coextensive with the class of functions $(1+z)^{1/2}f(z)$, where $f(z)$ is the value of a continued fraction of the above form, or of a terminating continued fraction of that form in which the last G_p may equal 1. (Received July 12, 1945.)

168. H. S. Wall: *Theorems on arbitrary J-fractions.*

Let $1/(b_1+z) - a_1^2/(b_2+z) - a_2^2/(b_3+z) - \cdots$ $(a_p \neq 0)$ be an arbitrary J-fraction. Let $x_p = X_p(z)$, $x_p = Y_p(z)$ be the solutions of the system $-a_{p-1}x_{p-1} + (b_p+z)x_p - a_px_{p+1} = 0$, $p=1, 2, 3, \cdots$ $(a_0=1)$ under the initial conditions $x_0 = -1$, $x_1 = 0$ and $x_0 = 0$, $x_1 = 1$, respectively. The *indeterminate case* or the *determinate case* holds according as both the series $\sum |X_p(0)|^2, \sum |Y_p(0)|^2$ converge or at least one diverges, respectively. It is shown that in the indeterminate case, if the J-fraction converges for a single value of z, it converges for every value of z to a meromorphic function. If the J-fraction is *positive definite*, the associated J-matrix has one or infinitely many bounded reciprocals for $I(z) > 0$ according as the determinate or the indeterminate case holds, respectively. Let $k_1, k_2, k_3, \cdots$ be numbers different from zero such that $\sum |k_{2p+1}|$ and $\sum |k_{2p+1}(k_2+k_4+ \cdots +k_{2p})^2|$ converge. If $\lim_{p=\infty} |k_2+k_4+ \cdots +k_{2p}| = \infty$, the continued fraction $1/k_1z + 1/k_2 + 1/k_3z + 1/k_4 + \cdots$ converges for every z to a meromorphic function or else to the constant ∞. If the above limit does not exist, or is finite, then the continued fraction diverges by oscillation for every z. (Received June 8, 1945.)

Applied Mathematics

169. Stefan Bergman: *The integration of equations of fluid dynamics in the three-dimensional case.*

The author describes methods for the determination of potentials of *three-dimensional* flow patterns which are of interest in the theory of turbines. In order to obtain an approximate potential of an axially symmetric flow of a given type defined in the domain D, he determines the complex potential $g(z)$, $z = x + iy$, of a two-dimensional flow in the meridian plane of D, that is, in the region which is the intersection of D with the plane $\phi = $ const. (r, θ, ϕ are the polar coordinates). Applying to $g(z)$ the operator introduced in Math. Zeit. vol. 24 (1926) pp. 641–669 he obtains a function which approximates the potential of the desired flow. (See, for example, the above paper, p. 655, where the potential of an axially symmetric flow in a turbine is given.) Using more complicated processes, potentials of general (not necessarily axis symmetric) flows can be obtained. These potential functions are used as first approximations to solutions of nonlinear equations of fluid dynamics. (Received July 30, 1945.)

170. Herbert Jehle: *A new approach to stellar statistics.*

A wave equation of the Schroedinger type, $[(\bar{\sigma}^2/2)\nabla^2 + (\bar{\sigma}/i)\partial/\partial t - c^2 - U] \cdot \{|\psi| \exp(iS/\bar{\sigma})\} = 0$, where $\bar{\sigma}$ is a constant, c the velocity of light and U the potential field, is known to admit a hydrodynamical interpretation: It splits into two real

equations, $-\sum \nabla_{\nu}(|\psi|^2\nabla_{\nu}S)+(\partial/\partial t)|\psi|^2=0$, a continuity equation with density $|\psi|^2$ and streaming velocity $-\nabla_{\nu}S$, and $(\bar{\sigma}^2/2|\psi|)\nabla^2|\psi|+\partial S/\partial t-c^2-U-\sum(\nabla_{\nu}S)^2/2=0$, a Hamilton-Jacobi equation or Bernouilli equation with an assumed pressure function $(\bar{\sigma}^2/2|\psi|)\nabla^2|\psi|$. An analysis of the motion of stars in a stellar system, treated as a hydrodynamical problem, shows that the above assumption about pressure function is plausible and fits the observed facts. Discrete effects (like quantum effects) cannot show up in this astronomical theory because observed hydrodynamical quantities in a stellar system are to be understood as averages taken over volumes containing many statistically independent elements (stars or star clusters), and they have to be confronted with a ψ function which is a superposition of many stationary ψ. Because of its mathematical simplicity this theory provides for an approach to problems such as transient solutions of the wave equation, selfconsistent steady and transient fields (that is, Poisson equation and hydrodynamical equations combined). (Received August 1, 1945.)

171. Arturo Rosenbluth and Norbert Wiener: *Mathematics of fibrillation and flutter in the heart.*

The known facts about the continuation and refractory period of a muscle fiber are used to explain the phenomena of flutter and fibrillation in the vertebra heart. A geometrical discussion is given of the flutter problem while the fibrillation problem is reduced to a statistical form. (Received July 20, 1945.)

172. H. E. Salzer: *Table of coefficients for repeated integration with differences.*

For functions tabulated at a uniform interval, formulas for k-fold integration, using advancing or backward differences, are obtained by integrating the Gregory-Newton advancing-difference interpolation formula or the Newton backward-difference formula. The quantities $G_n^{(k)}\equiv(1/n!)\times\int_0^1\cdots\int_0^p\int_0^p p(p-1)\cdots(p-n+1)(dp)^k$ and $H_n^{(k)}\equiv(1/n!)\times\int_0^1\cdots\int_0^p\int_0^p p(p+1)\cdots(p+n-1)(dp)^k$ are the coefficients of the nth advancing and backward differences respectively in the formulas $\int_{x_0}^{x_1}\cdots\int_{x_0}^{x}\int_{x_0}^{x}f(x)(dx)^k =h^k[f(x_0)/k!+\sum_{n=1}^m G_n^{(k)}\nabla^n f(x_0)]+Rm=h^k[f(x_0)/k!+\sum_{n=1}^m H_n^{(k)}\nabla^n f(x_0)]+R_m'$, where $x_1-x_0=h=$ the tabular interval. Previous tables (A. N. Lowan, H. E. Salzer, Journal of Mathematics and Physics vol. 22 (1943) pp. 49–50, and W. E. Milne, Amer. Math. Monthly vol. 40 (1933) pp. 322–327) furnish exact values for $k=1$, $n=1$ (1) 20 and $k=2$, $n=1$ (1) 7. The present table has exact values for $k=2$, $n=1$ (1) 20 and decimal values for $k=2$ (1) 6, $n=1$ (1) $22-k$. The quantities $G_n^{(2)}$ and $H_n^{(2)}$ are expressed in several ways as functions of $B_\nu^{(n)}(x)$, Bernoulli polynomials of order n and degree ν, where $t_ne^{xt}/(e^t-1)^n=\sum_{\nu=0}^\infty t^\nu B_\nu^{(n)}(x)/\nu!$, and were checked in terms of previously tabulated values of $B_\nu^{(n)}(x)$. Also a simple recursion formula for $G_n^{(k)}$ in terms of $G_n^{(k-1)}$ and $G_{n+1}^{(k-1)}$ (and similarly for $H_n^{(k)}$), valid for $k\geq2$, was used for computation when $k>2$. (Received July 7, 1945.)

GEOMETRY

173. P. O. Bell: *Power series expansions for the equations of a variety in hyperspace.*

For the study of the local properties of an arbitrary variety V_m in a linear space S_n a moving reference frame $F(x_0, x_1, \cdots, x_n)$ may be selected whose vertex x_0 is a generic point of V_m. The general projective homogeneous coordinates x_j^ρ of the points