ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

145. A. T. Brauer: *A problem of additive number theory and its application in electrical engineering.*

A set S of non-negative integers is called a basis of order 2 with regard to addition for the integer n if each non-negative integer $t \leqq n$ is the sum of two elements of S. Denote by $k = k_a(n)$ the smallest number of elements in any such basis for n, and for given k by $n = n_a(k)$ the greatest number for which a basis of k elements exists. Rohrbach [Math. Zeit. vol. 42 (1937) pp. 1–30] proved Schur's conjecture that $k_a(n) = O(n^{1/2})$. Moreover he proved that $.4992\ k^2 > n_a(k) > k^2/4 + 11k/6 - 237/12$. Similarly a basis with regard to subtraction is defined. For a basis with regard to addition and subtraction Rohrbach proved that $n_{a,s} \geqq k^2/4 + k/2 - 1$. In this paper, it is proved that for every k a basis with regard to subtraction can be determined for which $n_s \geqq k^2/4 + 7k/6 - 53/12$. For the construction of a special resistance, it was of interest to determine the exact value of $k_s(30)$. It is proved in this paper that $k_s(30) = 10$. Similarly $k_s(n)$ may be determined for every given n. (Received July 19, 1945.)

146. F. L. Brown: *A simplification of the postulates of tri-operational algebra.*

The author retains the postulate $10 = 1$ (juxtaposition denoting substitution) which is independent of the other assumptions of tri-operational algebra (cf. Reports of a Mathematical Colloquium, nos. 5–6, p. 13) and introduces the postulate $0f = f$ for each f. Then the postulates concerning 0, 1, and $-f$ (that is, $0 + f = f$, $1 \cdot f = f$, $f + (-f) = 0$ for each f) can be replaced by the weaker assumptions $0 + j = j$, $1 \cdot j = j$, and the existence of an element n such that $j + n = 0$ where j is the neutral element with respect to substitution ($jf = fj = f$). The postulate $0f = 0$ is independent of the other assumptions. While $0f$ is never equal to 1 or j, there does exist an algebra with five elements for one of which one has $0h = h \neq 0$. One may replace $0f = 0$ by $1f = 1$ in conjunction with $01 = 0$. (Received June 23, 1945.)

147. A. H. Copeland and Paul Erdös: *Note on normal numbers.*

D. G. Champernowne proved that if all of the positive integers are expressed in the base ten and arranged in order, the resulting sequence of digits when regarded as an infinite decimal constitutes a number which is normal in the sense of Borel. He conjectured that if the sequence of all integers were replaced by the sequence of primes, the corresponding decimal would be normal. In the present paper it is proved that a normal number can be produced in this manner by any sufficiently dense increasing sub-sequence of the positive integers. The validity of Champernowne's conjecture then follows from the fact that the primes are sufficiently dense. The same property holds for the sequence consisting of every integer which is the sum of two squares. These results are based on a concept of Besicovitch called (ϵ, k) normality. The paper concludes with two additional conjectures. (Received July 10, 1945.)

148. C. J. Everett: *Two representations for real numbers.*

Ideas on iterated functions due to B. H. Bissinger (*A generalization of continued fractions*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 868–876) are applied to *increasing* functions to obtain two quite different representations for real numbers with remarkable algebraic properties. (Received June 13, 1945.)

149. Bjarni Jónsson and Alfred Tarski: *On direct products of algebras.*

Consider algebras A closed under an operation $+$ and having a zero. A closed set $B\subseteq A$ with $0\in B$ is a *subalgebra* of A. The notion of a (*weak*) *direct product* of two or arbitrarily many subalgebras, $B\times C$ or $\prod_{i\in I}B_i$, is assumed to be known. B is a *factor* of A, $B\in\mathfrak{F}_A$, if $A=B\times C$ for some C. The *center* A^c is the set of all elements $a\in A$ with (i) $a+(x+y)=(a+x)+y=x+(a+y)$ for any x, $y\in A$, (ii) $a+\bar{a}=0$ for some $\bar{a}\in A$. *Theorems*: I. If (i) $\mathfrak{F}_A$ is a Boolean algebra under set inclusion, then (ii) A has the *refinement property*: $A=\prod_{i\in I}B_i=\prod_{j\in J}C_j$ implies $B_i=\prod_{j\in J}D_{ij}$, $C_j=\prod_{i\in I}D_{ij}$ for some subalgebras D_{ij} and all i, j; hence, (iii) A has at most one representation $A=\prod_{i\in I}B_i$ where B_i's are indecomposable. Conversely, I(ii) implies I(i). I(i) holds in all A where $x\in A^c\rightarrow x=0$ (examples: centerless groups, semigroups with $x+y=0\rightarrow x=y=0$, and lattices); also in algebras whose homomorphic images with two distinct elements are never commutative semigroups (examples: groups coinciding with their commutator groups, and A with a $u\in A$ such that $x\in A\rightarrow x+u=u$). II. If I(i) holds and the union of an increasing sequence of factors is always a factor of A, then A has just one representation I(iii); and conversely. II applies to all finite algebras satisfying I(i); also to algebras whose elements satisfy a finite chain condition. (Received July 27, 1945.)

150. Bjarni Jónsson and Alfred Tarski: *A generalization of Wedderburn's theorem.*

Using the terminology of the preceding abstract it is noted that A^c is always an Abelian group. *Theorem*: Let A be an algebra in which A^c is finite (or, more generally, in which every decreasing sequence of subgroups of A^c is finite, that is, where A^c is a torsion group of finite rank). Then (i) A has the *exchange property*: if $A=B\times C=\prod_{i\in I}D_i$, then $A=B\times\prod_{i\in I}D_i'$ where D_i' is a certain subalgebra of D_i for $i\in I$. Hence, (ii) A has the *weak refinement property*: if $A=\prod_{i\in I}B_i=\prod_{j\in J}C_j$, then, for some subalgebras B_{ij} and C_{ij}, $B_i=\prod_{j\in J}B_{ij}$, $C_j=\prod_{i\in I}C_{ij}$, and $B_{ij}\cong C_{ij}$ for $i\in I$ and $j\in J$. Consequently, (iii) if the factors B_i and C_j in (ii) are indecomposable, then they are pairwise isomorphic in a certain order. *Corollary*: Every finite algebra A (and, more generally, every algebra in which every decreasing sequence of congruence relations is finite) has, up to isomorphism, a unique representation as a direct product of finitely many indecomposable factors. (Received July 27, 1945.)

151. A. N. Milgram: *Cyclotomically saturated polynomials and tri-operational algebra.*

Let $f(x)$ be a polynomial whose coefficients are integers mod p where p is a prime number. Call $f(x)$ cyclotomically saturated if it has the property that for each irreducible polynomial $\phi(x)$, if $[\phi(x)]^\nu | f(x)$, then also $(x^{p^n}-x)^\nu | f(x)$ where n is the degree of $\phi(x)$. In tri-operational algebra (cf. Reports of a Mathematical Colloquium, nos. 5–6, p. 5) Menger raised the question: What polynomials with coefficients over the integers mod p have the property $f(x)|f(g(x))$ for each polynomial $g(x)$? The answer is: $f(x)|f(g(x))$ for each $g(x)$ if and only if $f(x)$ is cyclotomically saturated. (Received June 23, 1945.)

152. J. M. H. Olmsted: *Transfinite rationals.*

As suggested by the treatment of ratios by Eudoxus, two cardinal number pairs, (a, b) and (c, d), are defined to be equivalent if and only if for every pair of cardinal numbers, m and n, ma and nb have the same order relation as mc and nd. Addition, multiplication, division, and ordering are defined among the equivalence classes of cardinal number pairs, the resulting system being a lattice with familiar algebraic laws (for example, multiplication is distributive over addition, joins, and meets). This system is an extension of both the positive rational numbers and the cardinal numbers. Furthermore, it is the smallest extension subject to certain conditions. (Received June 4, 1945.)

153. Gordon Pall: *Hermitian quadratic forms in a quasi-field.*

Let F be a quasi-field, B_1 and B_2 nonsingular hermitian matrices of order $n-1$ in F, and let a be a nonzero scalar. Let there be given a transformation of $\bar{x}_0 a x_0 + \bar{x}' B_1 x$ into $\bar{x}_0 a x_0 + \bar{x}' B_2 x$. Then explicit transformations are constructed which replace B_1 by B_2. This is an extension of a similar result due to Witt for fields. (Received July 23, 1945.)

ANALYSIS

154. E. F. Beckenbach: *On a characteristic property of linear functions.*

Let there be given a class of real functions $\{f(x)\}$, defined and continuous in a closed and bounded interval, such that there is a unique member of the family which, at arbitrary distinct x_1, x_2 in the interval, takes on arbitrary values y_1, y_2 respectively. The class of linear functions is an example. It is shown that a real function $g(x)$, defined and continuous in the interval, is a member of $\{f(x)\}$ if and only if for each x_0 interior to the interval there exists an $h_0 = h_0(x_0)$ with $x_0 \pm h_0$ in the interval such that the member of $\{f(x)\}$ coinciding with $g(x)$ at $x_0 \pm h_0$ coincides with $g(x)$ also at x_0. (Received June 21, 1945.)

155. Stefan Bergman: *Pseudo harmonic vectors and their properties.*

The author applies the operator $\mathfrak{P}(f, \mathfrak{Q}, \mathfrak{T})$ introduced in Bull. Amer. Math. Soc. (vol. 49 (1943) p. 164) to complex functions $f = s^{(1)}(x, y) + is^{(2)}(x, y)$, for which $s_y^{(1)} = s_x^{(2)}$, $s_x^{(1)} = -s_y^{(2)} l(x)$ holds. Here $s_x^{(1)} = (\partial s^{(1)}/\partial x), \cdots$ and $l(x)$ is an analytic function of a real variable x. $\mathfrak{P}(s^{(1)} + is^{(2)}, \mathfrak{Q}, \mathfrak{T})$ yields a three-dimensional vector $\mathfrak{S}(X, Y, Z) = \mathfrak{S}^{(1)} + i\mathfrak{S}^{(2)} = \sum_{k=1}^{3} (S^{(k1)} + iS^{(k2)}) \mathfrak{i}_k$ for which curl $\mathfrak{S}^{(1)} = 0$, $S_X^{(11)}$