

$\alpha\gamma E\lambda\nu$ . 3. Construction:  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$  imply the existence of  $\xi$ ,  $\eta$  such that  $\lambda\mu K\lambda\xi$ ,  $\lambda\mu\nu K\lambda\xi\eta$ ,  $(\alpha\beta, \beta\gamma, \gamma\alpha)E(\xi\lambda, \lambda\eta, \eta\xi)$ , that is,  $\alpha\beta E\xi\lambda$ , and so on. 4. Linear uniqueness:  $\alpha\beta K\alpha\xi$  and  $\alpha\beta E\xi\alpha$  imply  $\xi = \beta$ . 5. Planar uniqueness:  $\alpha\beta\gamma K\alpha\beta\xi$ ,  $\alpha\gamma E\alpha\xi$ ,  $\beta\gamma E\beta\xi$  imply  $\xi = \gamma$ . 6. Equality of angles: If  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$ ,  $(\alpha\beta, \beta\gamma, \gamma\alpha) E (\lambda\mu, \mu\nu, \nu\lambda)$  then  $\alpha\xi\eta I\alpha\beta\gamma$  (that is,  $\alpha\xi K\alpha\beta$ ,  $\alpha\eta K\alpha\gamma$ ) implies the existence of  $\zeta$ ,  $\tau$  such that  $\lambda\xi\tau I\lambda\mu\nu$  and  $(\alpha\xi, \xi\eta, \eta\alpha) E (\lambda\xi; \zeta\tau, \tau\lambda)$ . Definitions: 1. The angle of the triangle  $\alpha\beta\gamma$  at  $\alpha$  is the class of triads  $\alpha\xi\eta$  such that  $\alpha\xi\eta I\alpha\beta\gamma$  and  $\alpha\beta\gamma K$ . 2. The angles of the triangles  $\alpha\beta\gamma$ ,  $\lambda\mu\nu$  at  $\alpha$  and  $\lambda$  are equal,  $\Delta\beta\gamma = \Delta\mu\nu$ , if and only if,  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$  and  $\alpha\xi\eta I\alpha\beta\gamma$  implies the existence of  $\zeta$ ,  $\tau$  as indicated under axiom 6. Reference is made to a paper previously reported in Bull. Amer. Math. Soc. vol. 43 (1937) p. 475. (Received May 18, 1945.)

136. A. R. Schweitzer: *A theory of congruence in the foundations of geometry*. III.

Relatively to the set of axioms in the preceding abstract, equality of dyads,  $\alpha\beta = \lambda\mu$ , "modulo  $K$ " and "modulo  $E$ " is defined to be  $\alpha\beta K\lambda\mu$  and  $\alpha\beta E\lambda\mu$  respectively. If  $E$  is replaced by  $K$  in the preceding axioms then axiom 3 is contradicted, axiom 4 is ineffective ("vacuously satisfied") and the remaining axioms are satisfied. If to the hypothesis of axiom 3 is added " $E \neq K$ " then for  $E = K$  axiom 3 is ineffective; thus a descriptive or metrical system results according as  $E = K$  or  $E \neq K$ . Correspondingly, equality of angles is defined modulo  $E$  ( $E \neq K$ ) and modulo  $K$  ( $E = K$ ). In the latter case angles are equal if and only if they coincide. Finally, an alternative set of axioms ( $E \neq K$ ) results from replacing the symbol  $(\xi\lambda, \lambda\eta, \eta\xi)$  by its conjugate  $(\lambda\xi, \xi\eta, \eta\lambda)$  in axiom 3, assuming that  $\alpha\beta K\alpha\beta$  implies  $\alpha\beta E\beta\alpha$ , and replacing the dyad  $\xi\alpha$  by its conjugate  $\alpha\xi$  in axiom 4. (Received May 18, 1945.)

#### TOPOLOGY

137. R. F. Arens: *The linear homogeneous continua of G. D. Birkhoff*.

A linear homogeneous continuum (LHC), in the sense of Birkhoff, is a linearly ordered set  $L$  in which every increasing (or decreasing) sequence of elements converges, and which can be placed in one-to-one, order preserving correspondence with any of its closed subintervals. Vasquez and Subieta (*Sobre los continuos homogéneos lineales de George D. Birkhoff*, Boletín de la Sociedad Matemática Mexicana vol. 1 (1944)) have given the first example of an LHC which is not an ordinary real closed interval. The present paper proves (1) if  $L$  is an LHC, then  $L^\omega$ , the class of all sequences in  $L$ , lexicographically ordered, is also an LHC, (2) if each well ordered subset of  $L$  has only countably many distinct elements, the same is true of  $L^\omega$ , and (3) if  $L$  is a real closed interval,  $L^\omega$  is not isomorphic to  $L$ . (Received May 10, 1945.)

138. R. H. Bing: *Concerning simple plane webs*.

It is shown that a necessary and sufficient condition that a compact plane continuous curve be a simple plane web is that it remain connected and locally connected on the omission of any countable set of points. Using this characterization of a simple plane web, the author considers some of its properties. (Received May 11, 1945.)

139. Salomon Bochner and Deane Montgomery: *Groups of differentiable and real or complex analytic transformations*.

The authors prove the following results: (1) If a Lie group acts on a manifold in

such a way that the transforming functions  $f(g; x)$  are continuous in  $g$  and  $x$  simultaneously and if for fixed  $g$  the functions  $f(g; x)$  are of class  $C^k$  (analytic) in  $x$ , then the functions  $f(g; x)$  are of class  $C^k$  (analytic) in the variables  $g$  and  $x$  simultaneously. (2) If a complex analytic group is compact it is abelian. From (1) and a known theorem is also obtained (3). If a compact group acts effectively on a connected manifold and if for each  $g$ ,  $f(g; x)$  is of class  $C^k$  ( $k \geq 1$ ) or analytic, then  $G$  is a Lie group and the functions  $f(g; x)$  are of class  $C^k$  or analytic in the variables  $(g, x)$  simultaneously. In (1) and (3) the manifold of course must be taken as of class  $C^k$  or analytic. The result (2) has been familiar to some mathematicians. (Received May 28, 1945.)

140. R. H. Fox: *Knots in 3-dimensional manifolds.*

Let  $M$  be a compact connected 3-dimensional manifold, let  $\gamma$  denote an element of  $\pi_1(M)$  and  $\bar{\gamma}$  the class of elements of  $\pi_1(M)$  conjugate to  $\gamma$ . Let  $K$  be any polygonal simple closed curve in  $M$  which represents  $\bar{\gamma}$  and does not intersect the boundary of  $M$ . Let  $R$  denote the nucleus of the injection  $\pi_1(M-K) \rightarrow \pi_1(M)$  and let  $[R]$  denote the commutator subgroup of  $R$ . It is proved that the group  $\omega(M, \bar{\gamma}) = \pi_1(M-K)/[R]$  is independent of the choice of representative  $K$ . The groups  $\omega(M, \bar{\gamma})$ , determined by the various classes  $\bar{\gamma}$  of  $\pi_1(M)$ , are invariants of the 3-dimensional manifold  $M$ . These groups are generally non-abelian and are independent of the homology groups. (Received May 16, 1945.)

141. Dean Montgomery: *Topological groups of differentiable transformations.*

If a locally compact group  $G$  acts on a manifold of class  $C^k$  in such a way that for a fixed  $g$  the transforming functions  $f(g; x)$  are of class  $C^k$  in  $x$ , then all partial derivatives with respect to the  $x$ 's of order  $k$  or less are simultaneously continuous in  $g$  and  $x$ . It follows that if  $G$  is compact and  $k \geq 1$  then  $G$  is a Lie group (considered in itself). The author had previously demonstrated this result in the analytic case by another method. (Received May 28, 1945.)

142. A. D. Wallace: *Extension sets. I.*

Although the results to be presented are valid for more general spaces, for simplicity the space  $H$  is assumed to be compact metric. By a subspace is meant a closed subset of  $H$ . A subspace  $M$  will be termed an *extension set* in dimension  $n$  (briefly, a  $J_n$ ) if for each subspace  $Y$  any mapping of  $Y \cdot M$  into  $S_n$  may be extended to  $Y$ . (1) Each  $J_n$  is a  $J_{n+1}$ . (2) The intersection of any collection of  $J_n$ 's is a  $J_n$ . A subset [subspace]  $Z$  is said to be  $n$ -connected [an  $n$ -continuum or a  $C_n$ ] if every mapping of  $Z$  into  $S_n$  is inessential on any compact subset contained in  $Z$ . (3) The intersection of an ordered (by inclusion) collection of  $C_n$ 's is a  $C_n$ . (4) The intersection of a  $C_n$  and a  $J_n$  is a  $C_n$ . If  $P$  is an admissible property for subspaces then a subspace is a  $P$ -endelement if it is contained in arbitrarily small neighborhoods with  $P$ -boundaries. (5) The intersection of an ordered collection of  $P$ -endelements is a  $P$ -endelement. (Received April 30, 1945.)

143. A. D. Wallace: *Extension sets. II.*

A point  $x$  is an  $n$ -cutpoint of a set  $X$  if  $X-x$  is not  $n$ -connected. A subspace will be termed a  $T_n$  if each of its subspaces is a  $C_n$ . (1) An  $n$ -cutpoint of a  $J_n$  is an  $n$ -cutpoint of  $H$ . (2) In order that a subspace be a  $J_n$  it is sufficient that its complement be the union of a collection of pairwise disjoint open sets whose boundaries are of type  $T_n$ . If  $f$  maps the subspace  $X$  into  $S_n$  then the subspace  $Y$  will be termed an es-

sential membrane for  $f$  provided that  $f$  can be extended to every proper subspace of  $Y$  but not to  $Y$ . (3) If  $M$  is a  $J_n$  containing the subspace  $X$ , and  $Y$  is an essential membrane for the mapping  $f$  of  $X$  into  $S_n$  [a  $C_n$  irreducible about  $X$ ] then  $Y$  is contained in  $M$ . (4) If  $X$  is a subspace not cut by any  $T_n$  and maximal relative to this property then  $X$  is a  $J_n$ . (Received April 30, 1945.)

144. A. D. Wallace: *Extension sets*. III.

Referring to definitions given in previous abstracts, the author proves five theorems. (1) Any  $T_n$  is a  $T_{n+1}$ . (2) If  $H$  is a  $C_n$  and for each subspace  $X$  each mapping of  $X$  into  $S_n$  has a unique essential membrane then  $H$  is a  $T_{n+1}$ . (3) If  $X$  is a subspace and  $Y$  is a  $C_n$  irreducible about  $X$  [an essential membrane for some mapping of  $X$  into  $S_n$ ] then no point of  $Y-X$  is a  $T_n$ -endelement. (4) If  $H$  is a  $C_n$  and  $Z$  is the set of all points not  $T_n$ -endelements then  $Z$  is  $n$ -connected. (5) If  $H$  is a  $C_n$  and  $X$  is a  $C_n$ -endelement then  $X$  is a  $C_n$ . Many of the results presented are valid if  $H$  is a compact Hausdorff space. In some cases  $S$  may be replaced by a space having the neighborhood extension property. In the paper reference will be made to the work of Ayres, Borsuk, Eilenberg, Hurewicz, and G. T. Whyburn. (Received April 30, 1945.)