

There are indications that these coefficients can be used to extend the solution of a second order partial differential equation of the form  $u_{xx} = u_t + \psi(u, x, t)$ , provided that it is known at a rectangular array of points in the  $x, t$ -plane and at two other points in the next row of values of  $t$ . (Received April 30, 1945.)

### GEOMETRY

129. P. O. Bell: *Metric properties of a class of quadratic differential forms.*

In the present paper a new invariant quadratic differential form  $\Omega$  is geometrically defined for a general pair of surfaces  $S, S'$  whose corresponding points  $x, x'$  determine the metric normal to  $S$  at  $x$ . The ratio of the form  $\Omega$  to the first fundamental form  $ds^2$  of  $S$ , in which  $\Omega$  and  $ds^2$  are defined for a common arc element of  $S$  at  $x$ , is found to be independent of the direction of the element if and only if the surface  $S'$  is the locus of the center of mean curvature of  $S$ ; the ratio thus determined is the Gaussian curvature  $K$  of  $S$  at  $x$ . It is proved that the form  $\Omega$  for an arbitrary arc element is identical with the form  $Kds^2$  for either "conjugate" element if and only if the surface  $S'$  is the plane net at infinity. The principal directions at  $x$  of the tensor whose components are the coefficients of the form  $\Omega$  are the classical principal directions of  $S$  at  $x$  for an arbitrary choice of  $S'$ . Finally, the net of lines of mean-curvature of  $S$  and the mean-conjugate net of  $S$  are characterized as integral nets of equations of the form  $\Omega = 0$ , for suitable selections of  $S'$ . The author employs dual systems of linear equations of the first order with the use of a tensor notation. (Received May 19, 1945.)

130. L. M. Blumenthal: *Characterization of  $\phi$ -spherical subsets and pseudo sets.*

The class of  $\phi$ -spherical spaces is defined by four metric postulates involving an arbitrary function  $\phi$ . The class contains, for example, those spaces derived from the surface of the ordinary sphere in euclidean  $(n+1)$ -space by making it metric with respect to geodesic (shorter arc) distance and with respect to euclidean (chord) distance. The paper develops the metric geometry of this class of spaces and obtains the metric characterizations of the subsets and pseudo sets. (The paper is to appear (in Spanish) in *Revista de Universidad Nacional de Tucumán*, Ser. A. vol. 5 under the title *La caracterización métrica de espacios  $\phi$ -esféricos.*) (Received May 18, 1945.)

131. L. M. Blumenthal: *Metric study of elliptic spaces.*

Among the properties of elliptic spaces which make inapplicable the procedures usually employed in a metric study are (1) the "unusual" character of the locus of points equidistant from two points, (2) the lack of free movability in the large, (3) the necessity of distinguishing between "contained in" and "congruently contained in," (4) the notions of dependence and independence of subsets, if defined in the ordinary way, are not metrically invariant, and (5) the abnormal behavior with respect to equilateral subsets. The writer presents a new approach in which the metrically invariant notions of *relative independence* (*dependence*) and *class independence* (*dependence*) play fundamental roles. In terms of these notions the elliptic line is metrically defined. The extension to plane, and so on, is then possible in conventional manner. Necessary and sufficient conditions (of a *quasi-metric* nature) in order that congruent subsets of an elliptic space be superposable are obtained. (Received May 19, 1945.)

132. S. S. Chern: *Characteristic classes of Hermitian manifolds. I.*

With a manifold having a complex analytic structure is associated in an intrinsic way the fibre bundle of its complex tangent vectors. By retraction it can be assumed in most problems that the fibres are spheres of dimension  $2n - 1$  on which operates the complex unitary group in  $n$  variables. The following theorems are proved: (1) This fibre bundle can be imbedded in the sense of Whitney-Steenrod in the Grassmann manifold of all  $n$ -dimensional (complex dimension) linear spaces through the origin of a complex Euclidean space of dimension  $n + N$ , provided that  $n \leq N$ . (2) The imbedding is defined up to a homotopy in the sense that two such fibre bundles are equivalent when and only when the maps in the Grassmann manifold are homotopic. (3) The characteristic cohomology classes of the manifold, being the inverse images under this mapping of the cohomology classes of dimension less than or equal to  $2n$  of the Grassmann manifold, can be obtained from  $n$  basic characteristic classes under operations of the cohomology ring. These  $n$  basic classes are the ones analogous to the Stiefel-Whitney classes in the case of real vector fields over real differentiable manifolds, the difference being that they are always classes with integral coefficients. (Received May 25, 1945.)

133. S. S. Chern: *Characteristic classes of Hermitian manifolds. II.*

In the previous paper the author proved that all characteristic classes of a complex analytic manifold of dimension  $n$  can be derived from  $n$  basic ones by operations of the cohomology ring. In case the manifold carries an Hermitian metric which is everywhere regular, these  $n$  classes can be expressed in the sense of de Rham in terms of simple invariant differential forms constructed from the Hermitian metric. One of these relations (namely, that corresponding to the highest dimension) is the formula of Gauss-Bonnet as generalized by Allendoerfer-Weil. If the manifold is the complex projective space with the elliptic Hermitian metric, certain formulas of Cartan and Wirtinger can be derived from these formulas as simple consequences. (Received May 25, 1945.)

134. H. F. DeBaggis: *A reduction of the postulational basis of non-euclidean geometry.*

In order to prove the law of Pasch in Bolyai-Lobachevsky geometry Jenks (Reports of a Mathematical Colloquium vol. 2, p. 10) introduced a postulate VIII which Landin (ibid. vols. 5-6, p. 57) reformulated as follows: If  $B$  is between  $A$  and  $C$ , and if  $a$  and  $c$  are non-intersecting lines on  $A$  and  $C$ , respectively, and  $d$  is a line intersecting  $a$  and  $c$ , then  $d$  intersects every line on  $B$  which does not intersect  $a$  and  $c$ . This statement can be derived from Jenks' postulates I, V-VII (ibid. vol. 1, p. 46) and the author's postulates II', III', IV' (Bull. Amer. Math. Soc., abstract 51-5-97) which are weaker than Jenks' postulates II, III, IV. From I, II'-IV', V-VII one can deduce the entire theory of linear and planar order except the theorem that for each two points  $P$  and  $Q$  there exists a point  $R$  such that  $Q$  is between  $P$  and  $R$ . This theorem can be obtained if IV' is replaced by Jenks' original IV. (Received April 3, 1945.)

135. A. R. Schweitzer: *A theory of congruence in the foundations of geometry. II.*

Supplementing his descriptive axioms in terms of a relation  $\alpha\beta K\lambda\mu$  (Trans. Amer. Math. Soc. vol. 10 (1909) p. 309) the author constructs axioms for metrical geometry in terms of a relation  $\alpha\beta E\lambda\mu$  as follows: 1. External transitivity:  $\alpha\beta E\lambda\mu$  and  $\lambda\mu E\nu\omega$  imply  $\alpha\beta E\nu\omega$ . 2. Internal transitivity:  $\alpha\beta K\beta\gamma$ ,  $\lambda\mu K\mu\nu$ ,  $\alpha\beta E\lambda\mu$  and  $\beta\gamma E\mu\nu$  imply

$\alpha\gamma E\lambda\nu$ . 3. Construction:  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$  imply the existence of  $\xi$ ,  $\eta$  such that  $\lambda\mu K\lambda\xi$ ,  $\lambda\mu\nu K\lambda\xi\eta$ ,  $(\alpha\beta, \beta\gamma, \gamma\alpha)E(\xi\lambda, \lambda\eta, \eta\xi)$ , that is,  $\alpha\beta E\xi\lambda$ , and so on. 4. Linear uniqueness:  $\alpha\beta K\alpha\xi$  and  $\alpha\beta E\xi\alpha$  imply  $\xi = \beta$ . 5. Planar uniqueness:  $\alpha\beta\gamma K\alpha\beta\xi$ ,  $\alpha\gamma E\alpha\xi$ ,  $\beta\gamma E\beta\xi$  imply  $\xi = \gamma$ . 6. Equality of angles: If  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$ ,  $(\alpha\beta, \beta\gamma, \gamma\alpha) E (\lambda\mu, \mu\nu, \nu\lambda)$  then  $\alpha\xi\eta I\alpha\beta\gamma$  (that is,  $\alpha\xi K\alpha\beta$ ,  $\alpha\eta K\alpha\gamma$ ) implies the existence of  $\zeta$ ,  $\tau$  such that  $\lambda\xi\tau I\lambda\mu\nu$  and  $(\alpha\xi, \xi\eta, \eta\alpha) E (\lambda\xi; \zeta\tau, \tau\lambda)$ . Definitions: 1. The angle of the triangle  $\alpha\beta\gamma$  at  $\alpha$  is the class of triads  $\alpha\xi\eta$  such that  $\alpha\xi\eta I\alpha\beta\gamma$  and  $\alpha\beta\gamma K$ . 2. The angles of the triangles  $\alpha\beta\gamma$ ,  $\lambda\mu\nu$  at  $\alpha$  and  $\lambda$  are equal,  $\Delta\beta\gamma = \Delta\mu\nu$ , if and only if,  $\alpha\beta\gamma K$ ,  $\lambda\mu\nu K$  and  $\alpha\xi\eta I\alpha\beta\gamma$  implies the existence of  $\zeta$ ,  $\tau$  as indicated under axiom 6. Reference is made to a paper previously reported in Bull. Amer. Math. Soc. vol. 43 (1937) p. 475. (Received May 18, 1945.)

136. A. R. Schweitzer: *A theory of congruence in the foundations of geometry*. III.

Relatively to the set of axioms in the preceding abstract, equality of dyads,  $\alpha\beta = \lambda\mu$ , "modulo  $K$ " and "modulo  $E$ " is defined to be  $\alpha\beta K\lambda\mu$  and  $\alpha\beta E\lambda\mu$  respectively. If  $E$  is replaced by  $K$  in the preceding axioms then axiom 3 is contradicted, axiom 4 is ineffective ("vacuously satisfied") and the remaining axioms are satisfied. If to the hypothesis of axiom 3 is added " $E \neq K$ " then for  $E = K$  axiom 3 is ineffective; thus a descriptive or metrical system results according as  $E = K$  or  $E \neq K$ . Correspondingly, equality of angles is defined modulo  $E$  ( $E \neq K$ ) and modulo  $K$  ( $E = K$ ). In the latter case angles are equal if and only if they coincide. Finally, an alternative set of axioms ( $E \neq K$ ) results from replacing the symbol  $(\xi\lambda, \lambda\eta, \eta\xi)$  by its conjugate  $(\lambda\xi, \xi\eta, \eta\lambda)$  in axiom 3, assuming that  $\alpha\beta K\alpha\beta$  implies  $\alpha\beta E\beta\alpha$ , and replacing the dyad  $\xi\alpha$  by its conjugate  $\alpha\xi$  in axiom 4. (Received May 18, 1945.)

#### TOPOLOGY

137. R. F. Arens: *The linear homogeneous continua of G. D. Birkhoff*.

A linear homogeneous continuum (LHC), in the sense of Birkhoff, is a linearly ordered set  $L$  in which every increasing (or decreasing) sequence of elements converges, and which can be placed in one-to-one, order preserving correspondence with any of its closed subintervals. Vasquez and Subieta (*Sobre los continuos homogéneos lineales de George D. Birkhoff*, Boletín de la Sociedad Matemática Mexicana vol. 1 (1944)) have given the first example of an LHC which is not an ordinary real closed interval. The present paper proves (1) if  $L$  is an LHC, then  $L^\omega$ , the class of all sequences in  $L$ , lexicographically ordered, is also an LHC, (2) if each well ordered subset of  $L$  has only countably many distinct elements, the same is true of  $L^\omega$ , and (3) if  $L$  is a real closed interval,  $L^\omega$  is not isomorphic to  $L$ . (Received May 10, 1945.)

138. R. H. Bing: *Concerning simple plane webs*.

It is shown that a necessary and sufficient condition that a compact plane continuous curve be a simple plane web is that it remain connected and locally connected on the omission of any countable set of points. Using this characterization of a simple plane web, the author considers some of its properties. (Received May 11, 1945.)

139. Salomon Bochner and Deane Montgomery: *Groups of differentiable and real or complex analytic transformations*.

The authors prove the following results: (1) If a Lie group acts on a manifold in