

three-dimensional projective space which satisfy the condition $\beta\psi^3 - \gamma\phi^3 = 0$. The subclass of these surfaces which also satisfy the condition $\phi\psi k^2 = 9\beta\gamma$ with a constant k plays an important role. Applications are made to conjugal quadrics, D_k quadrics and curves, asymptotic section quadrics, chord section quadrics, the Segre-Darboux conjugate nets, deformable curves, projective curves, axial quadrics, and other projective differential concepts. (Received September 29, 1944.)

LOGIC AND FOUNDATIONS

282. A. E. Meder and Elizabeth Hallett: *The equipollence of two systems of axioms.*

It is proved that axioms 1-5 of the set given by Weisner (Trans. Amer. Math. Soc. vol. 38 (1935) pp. 474-484) for a "hierarchy" and the set of axioms given by Klein-Barmen (Math. Ann. vol. 111 (1935) pp. 596-621) for a "Verband" are equipollent, and that Weisner's axiom 6 is not derivable from Klein's axioms. (Received September 30, 1944.)

STATISTICS AND PROBABILITY

283. L. A. Aroian: *The frequency function of the product of two normally distributed variables.*

Let $z = xy/\sigma_1\sigma_2$, x and y normally distributed, the means be denoted by \bar{X} , \bar{Y} , standard deviations by σ_1 , σ_2 , coefficient of correlation by r_{xy} , and $\rho_1 = \bar{X}/\sigma_1$, $\rho_2 = \bar{Y}/\sigma_2$. C. C. Craig (Annals of Mathematical Statistics vol. 7 (1936) pp. 1-15) has obtained the probability function of the product as the difference of two integrals and also as the sum of an infinite series of Bessel functions of the second kind. The author proves that as ρ_1 and ρ_2 approach infinity in any manner the distribution of z approaches normality, $-1 \leq r \leq 1$. The theorem holds in addition under the circumstances $\rho_1 \rightarrow \infty$, $\rho_2 \rightarrow -\infty$; $\rho_1 \rightarrow -\infty$, $\rho_2 \rightarrow -\infty$; $\rho_1 \rightarrow -\infty$, $\rho_2 \rightarrow \infty$; with $-1 < r < 1$. The distribution of z approaches normality, further, if ρ_1 is constant, $\rho_2 \rightarrow \infty$, $-1 \leq r \leq 1$; and the same variations in ρ_1 and ρ_2 apply as in the previous theorem. The special case $r=0$ is discussed in some detail. When ρ_1 and ρ_2 are small, the Gram-Charlier Type A series and the Pearson Type III function are excellent approximations in the proper regions. To determine the numerical values of the exact probability function of z , the author uses mechanical quadrature to evaluate the two integrals, since the series development converges very slowly even for values of ρ_1 and ρ_2 as small as 2. (Received September 25, 1944.)

TOPOLOGY

284. R. F. Arens: *Topologies for the class of continuous functionals on a completely regular space.*

Among the topologies that can be introduced into a class C of continuous mappings of one topological space A into another, B , those which are "admissible" are of especial interest. A topology for C is admissible if the expression $f(x)$, where f is an element of C and x is a point in A , depends continuously on f and x simultaneously (cf. abstract 49-1-89). The paper contains a proof that a weakest admissible topology for C always exists when A is completely regular and locally bicomact. (One topology is weaker than another if the open sets of the former are included among the open sets of the latter.) On the other hand, if A is completely regular and B contains a simple arc (in particular, if B is a real interval) and C is known to have a weakest admissible

topology, it follows that A is locally bicomact. This result is interesting because it was once conjectured that a weakest admissible topology would always exist whether A were locally bicomact or not, and because it provides a new characterization for locally bicomact spaces. (Received August 28, 1944.)

285. P. J. Kelly: *Some properties of a certain interchange type of self-isometry.*

An isometric mapping of a set on itself, not the identity, which leaves each element invariant or else interchanges it with another is defined as a self-isometry of order two. It is shown that if each of two finite, isometric sets has the above type of self-isometry, and if the non-smaller of the two subsets left respectively invariant does not have this type of self-isometry, then there is an isometry of the original sets which maps one invariant subset on the other. This result is applied to prove that if the cartesian product sets E^2 and F^2 are formed from finite metric sets E and F , and are metrized by one of a certain class of functions, then E^2 isometric to F^2 implies E isometric to F provided that one of the diagonal sets of E^2 and F^2 fails to have a self-isometry of order two. (Received September 20, 1944.)

286. A. H. Stone: *On the normality of product spaces.* Preliminary report.

It is proved that a necessary and sufficient condition for a product of separable metric spaces to be normal is that all but a countable number (at most) of the factor spaces be compact. (Received August 7, 1944.)

287. P. A. White: *On regular transformations.*

A transformation $T(K) = K'$, where K is a compact metric space and T is continuous and single-valued, is called n -regular if for any sequence of points (y_i) of K' converging to y , one has $(T^{-1}(y_i)) \rightarrow T^{-1}(y)$, r -regularly for all $r \leq n$. (For definitions of r -regular convergence see G. T. Whyburn, *On sequences and limiting sets*, Fund. Math. vol. 25 (1935).) It is shown that if T is n -regular, then the n -dimensional Betti group of K is the direct sum of two groups, one of which is isomorphic with the n -dimensional Betti group of K' , while the other is isomorphic with the n -dimensional Betti group of $T^{-1}(y)$ relative to K for any point $y \in K'$. It is also shown that the property of being an l^n (locally connected for all dimensions not greater than n) is invariant under an $(n-1)$ -regular transformation, and that this property is invariant under the inverse of an n -regular transformation. Finally it is shown that under n -regular transformations the property of being an n -dimensional closed Cantorian manifold is invariant provided the dimension is not increased and that the transform contains more than one point. (Received September 23, 1944.)

288. G. W. Whitehead: *On products in homotopy groups.*

This paper is devoted to an investigation of the multiplication in homotopy groups defined by J. H. C. Whitehead (Ann. of Math. vol. 42 (1941) p. 411). The principal tool is a new product, depending on four elements $\alpha \in \pi_p(X)$, $\beta \in \pi_q(X)$, $\phi \in \pi_{q-1}(F)$, and $\psi \in \pi_{p-1}(F')$, where F and F' are certain function spaces over X depending on α and β , and π_p denotes the p th homotopy group. The generalized products of J. H. C. Whitehead are characterized in terms of the ordinary products and certain isomorphisms between the homotopy groups of X and F . As a corollary to a general

theorem on the structure of the function space X^S it is shown that the "Einhängung" (Freudenthal, *Compositio Math.* vol. 5 (1937) pp. 299–314) of a product is always inessential. Finally it is shown that if α is an element of a certain subgroup of $\pi_{p+q-1}(S^p)$ (the whole group if $q \leq p$), and β, β' are elements of $\pi_p(X)$, then $(\beta + \beta')\alpha = [\beta\gamma, \beta'\delta] + \beta\alpha + \beta'\alpha$, where γ and δ are elements of $\pi_p(S^p)$ and $\pi_q(S^p)$ depending only on α . (Received August 23, 1944.)