x-axis at the time $t=0$. Let $F(x, t)$ denote the resulting temperature distribution at the time $t>0$ according to the equation $4\partial F/\partial t = \partial^2 F/\partial x^2$. For a fixed t, say $t=1/2$, $F(x, t)$ depending linearly on the parameters $\{f_n\}$ is now used as a family of interpolating functions. Indeed, for a given sequence of equidistant ordinates $\{y_n\}$ $(-\infty < n < \infty)$, the interpolation problem $F(n, t) = y_n$ $(-\infty < n < \infty)$ admits the following explicit solution: (1) $F(x, t) = \sum_{n=-\infty}^{\infty} y_n N(x-n, t)$. Here $N(x, t) = (1/2\pi) \cdot \int_{-\infty}^{\infty} \{\psi(u)/\phi(u)\} \cos ux \, du$ where $\psi(u) = (\sin 2^{-1}u/2^{-1}u)^2 \exp\{-t(2^{-1}u)^2\}$, $\phi(u) = \sum_{k=-\infty}^{\infty} \psi(u+2\pi k)$. For $t=0$, $N(x, 0) = 1 - |x|$ $(-1 \leqq x \leqq 1)$, $N(x, 0) = 0$ $(x>1$ or $x<-1)$, and (1) reduces to the *linear interpolation* of the ordinates $\{y_n\}$. For $t\rightarrow\infty$, $N(x, \infty) = \sin \pi x/\pi x$ and (1) reduces to the *cardinal interpolation series* (2) $F(x, \infty) = \sum_n y_n \{\sin \pi(x-n)/\pi(x-n)\}$. For a fixed finite t, such as $t=1/2$, formula (1) combines the smooth character of (2) $(t=\infty)$ with the computational advantages of linear interpolation $(t=0)$. The computational advantage of a finite t arises from the exponential damping of $N(x, t)$ as compared with the slow damping of $\sin \pi x/\pi x$. (Received October 2, 1944.)

279. I. J. Schoenberg: *On smoothing and subtabulation of empirical functions by means of heat-flow.* II.

Given a sequence of ordinates to be smoothed and subtabulated; the parameters $\{f_n\}$ of the interpolating function $F(x, t)$ of the previous paper are now determined so as to minimize $S = \sum_{n=-\infty}^{\infty} \{F(n, t) - y_n\}^2 + \epsilon \sum_{n=-\infty}^{\infty} (f_n - y_n)^2$, where ϵ is a positive smoothing parameter. For $\epsilon = 0$ the previous interpolation problem results. For $\epsilon = \infty$, $f_n = y_n$ and $F(x, t)$ is the smoothed version by heat-flow of the polygonal line $F(x, 0)$ of vertices (n, y_n). A compromise between strict interpolation $(\epsilon=0)$ and pure smoothing $(\epsilon=\infty)$ gives the explicit solution (1) $F(x, t, \epsilon) = \sum_{n=-\infty}^{\infty} y_n N(x-n, t, \epsilon)$, where $N(x, t, \epsilon) = (2\pi)^{-1} \int_{-\infty}^{\infty} \{(\epsilon + \phi(u))/(\epsilon + \phi^2(u))\} \psi(u) \cos ux \, du$. Clearly $N(x, t, 0)$ is identical with $N(x, t)$ of the previous paper. Eight-place tables have just been computed on punched card machines for the function $N(x, t, \epsilon)$ and its derivatives $N_x'(x, t, \epsilon)$, $N_x''(x, t, \epsilon)$ for $t=1/2$, $\epsilon = 0, 0.1, 0.2, \cdots, 0.9, 1.0$ and the range $-18.5 \leqq x \leqq 18.5$ (step 0.1) outside of which these functions vanish to 8 places. On increasing the smoothing parameter ϵ, the approximation (1) becomes smoother in the following sense: If $\sum y_n < \infty$ all integrals $\int_{-\infty}^{\infty} \{F^{(k)}(x, t, \epsilon)\}^2 dx$ $(k=0, 1, 2, \cdots)$ exist and each is a monotone decreasing function of ϵ in the range $0 \leqq \epsilon < \infty$. This procedure is being applied to certain empirical functions for which very smooth tables are required. (Received October 2, 1944.)

280. Alexander Weinstein and J. R. Pounder: *On two elementary problems of mechanics and electromagnetic theory.*

It is shown that the problem of the motion of a heavy particle on a rotating earth and the problem of the motion of a point charge in uniform electric and magnetic fields are mathematically equivalent, except for a change of axes; whereas they are usually treated by different methods. (Received September 29, 1944.)

Geometry

281. J. E. Wilkins: *A special class of surfaces in projective differential geometry.* II.

In this paper, which is a sequel to one appearing under the same title (Duke Math. J. vol. 10 (1943) pp. 667–675), a more intensive study is made of surfaces in three-dimensional projective space which satisfy the condition $\beta\psi^3 - \gamma\phi^3 = 0$. The subclass of these surfaces which also satisfy the condition $\phi\psi k^2 = 9\beta\gamma$ with a constant k plays an important role. Applications are made to conjugal quadrics, D_k quadrics and curves, asymptotic section quadrics, chord section quadrics, the Segre-Darboux conjugate nets, deformable curves, projective curves, axial quadrics, and other projective differential concepts. (Received September 29, 1944.)

Logic and Foundations

282. A. E. Meder and Elizabeth Hallett: *The equipollence of two systems of axioms.*

It is proved that axioms 1–5 of the set given by Weisner (Trans. Amer. Math. Soc. vol. 38 (1935) pp. 474–484) for a "hierarchy" and the set of axioms given by Klein-Barmen (Math. Ann. vol. 111 (1935) pp. 596–621) for a "Verband" are equipollent, and that Weisner's axiom 6 is not derivable from Klein's axioms. (Received September 30, 1944.)

Statistics and Probability

283. L. A. Aroian: *The frequency function of the product of two normally distributed variables.*

Let $z = xy/\sigma_1\sigma_2$, x and y normally distributed, the means be denoted by $\overline{X}$, $\overline{Y}$, standard deviations by σ_1, σ_2, coefficient of correlation by r_{xy}, and $\rho_1 = \overline{X}/\sigma_1$, $\rho_2 = \overline{Y}/\sigma_2$. C. C. Craig (Annals of Mathematical Statistics vol. 7 (1936) pp. 1–15) has obtained the probability function of the product as the difference of two integrals and also as the sum of an infinite series of Bessel functions of the second kind. The author proves that as ρ_1 and ρ_2 approach infinity in any manner the distribution of z approaches normality, $-1 \leqq r \leqq 1$. The theorem holds in addition under the circumstances $\rho_1 \to \infty$, $\rho_2 \to -\infty$; $\rho_1 \to -\infty$, $\rho_2 \to -\infty$; $\rho_1 \to -\infty$, $\rho_2 \to \infty$; with $-1 < r < 1$. The distribution of z approaches normality, further, if ρ_1 is constant, $\rho_2 \to \infty$, $-1 \leqq r \leqq 1$; and the same variations in ρ_1 and ρ_2 apply as in the previous theorem. The special case $r = 0$ is discussed in some detail. When ρ_1 and ρ_2 are small, the Gram-Charlier Type A series and the Pearson Type III function are excellent approximations in the proper regions. To determine the numerical values of the exact probability function of z, the author uses mechanical quadrature to evaluate the two integrals, since the series development converges very slowly even for values of ρ_1 and ρ_2 as small as 2. (Received September 25, 1944.)

Topology

284. R. F. Arens: *Topologies for the class of continuous functionals on a completely regular space.*

Among the topologies that can be introduced into a class C of continuous mappings of one topological space A into another, B, those which are "admissible" are of especial interest. A topology for C is admissible if the expression $f(x)$, where f is an element of C and x is a point in A, depends continuously on f and x simultaneously (cf. abstract 49-1-89). The paper contains a proof that a weakest admissible topology for C always exists when A is completely regular and locally bicompact. (One topology is weaker than another if the open sets of the former are included among the open sets of the latter.) On the other hand, if A is completely regular and B contains a simple arc (in particular, if B is a real interval) and C is known to have a weakest admissible