275. Otto Szász: *On Lebesgue summability and its generalization to integrals.*

Hardy and Littlewood proved that $(C, -\alpha)$ summability for some positive $\alpha < 1$ implies Lebesgue summability. The author proves the following generalization: if a series is summable $(C, 1-\alpha)$ and the absolute values of the $(C, -\alpha)$ means are bounded $(C, 1)$, then the series is Lebesgue summable. This summation method is then generalized to integrals and to a general class of trigonometric series. (Received September 29, 1944.)

Applied Mathematics

276. Stefan Bergman: *Representation of potentials of electric charge distributions.*

The author investigates the harmonic functions $V(x_k)$, $k=1, 2, 3$, obtained as potentials of charges of the linear density $F(\zeta)$ along curves $[x_k^* = \xi_k(\zeta),\ k=1, 2, 3,\ \zeta_0 \leqq \zeta \leqq \zeta_1]$, where F and the ξ_k are rational functions of ζ. $V(\zeta_0, \zeta_1; x_k)$ is a hyperelliptic integral, $\int_{\zeta_0}^{\zeta_1} F(\zeta)\left[\prod_\nu(\zeta - a_\nu)\right]^{-1/2} d\zeta$, where the a_ν are algebraic functions of the x_k. Generalizing his previous considerations (see Math. Ann. vol. 101 (1929) p. 534 and Bull. Amer. Math. Soc. vol. 49 (1943) p. 163) and using classical results, the author studies the $V(\zeta_0, \zeta_1; x_k)$ considered as functions of the x_k. Every V can be represented as a sum of the integrals of the first, second and third kind. The periods $\omega_{\alpha\beta}(x_k)$ of the integrals of the first kind are entire functions of the a_ν. The periods of the integrals of the second and third kinds can be expressed in a closed form using θ-functions in terms of the $\omega_{\alpha\beta}(x_k)$. Every $V(\zeta_0, \zeta_1; x_k)$ can be expressed in a closed form using θ-functions in terms of the $\omega_{\alpha\beta}(x_k)$ and the integrals of the first kind. Analogous results are obtained for the potentials V considered as functions of the x_k and some additional parameters Y_n, say the coefficients of the function $\xi_k(\zeta)$. (Received September 29, 1944.)

277. H. W. Eves: *Calculating machine computation forms for the three-point problem on a rectangular coordinate system.*

The three-point problem of surveying has been solved by plane table methods, mechanical methods, trigonometrical methods, geometrical construction methods, and coordinate geometry methods. In view of the general trend to tie surveys into the various state plane coordinate systems, and because of the increasing adoption by engineering concerns of calculating machine procedures to replace the former logarithmic ones, the last, or coordinate geometry methods, are considered the most important. In this paper four different geometrical construction solutions are utilized in building up four corresponding calculating machine forms for finding the coordinates of the observation point in terms of the coordinates of the three observed points and the two observed angles. These forms do not exhibit any very great differences in length or simplicity, running from one of forty entries to one of fifty-six entries, and involving more or less the same combinations of the given elements. One of the forms is essentially that developed and used by the Tennessee Valley Authority. The others are believed to be new. (Received September 11, 1944.)

278. I. J. Schoenberg: *An interpolation formula derived from heat-flow.* I.

Let $\{f_n\}$ $(-\infty < n < \infty)$ be a real sequence. Consider the function $F(x, 0)$ whose graph is the polygonal line of vertices (n, f_n) as a temperature distribution along the

x-axis at the time $t=0$. Let $F(x, t)$ denote the resulting temperature distribution at the time $t>0$ according to the equation $4\partial F/\partial t=\partial^2 F/\partial x^2$. For a fixed t, say $t=1/2$, $F(x, t)$ depending linearly on the parameters $\{f_n\}$ is now used as a family of interpolating functions. Indeed, for a given sequence of equidistant ordinates $\{y_n\}$ $(-\infty<n<\infty)$, the interpolation problem $F(n, t)=y_n$ $(-\infty<n<\infty)$ admits the following explicit solution: (1) $F(x, t)=\sum_{n=-\infty}^{\infty} y_n N(x-n, t)$. Here $N(x, t)=(1/2\pi)\cdot\int_{-\infty}^{\infty}\{\psi(u)/\phi(u)\}\cos ux\, du$ where $\psi(u)=(\sin 2^{-1}u/2^{-1}u)^2 \exp\{-t(2^{-1}u)^2\}$, $\phi(u)=\sum_{k=-\infty}^{\infty}\psi(u+2\pi k)$. For $t=0$, $N(x, 0)=1-|x|$ $(-1\leqq x\leqq 1)$, $N(x, 0)=0$ ($x>1$ or $x<-1$), and (1) reduces to the *linear interpolation* of the ordinates $\{y_n\}$. For $t\rightarrow\infty$, $N(x, \infty)=\sin \pi x/\pi x$ and (1) reduces to the *cardinal interpolation series* (2) $F(x, \infty)=\sum_n y_n\{\sin \pi(x-n)/\pi(x-n)\}$. For a fixed finite t, such as $t=1/2$, formula (1) combines the smooth character of (2) $(t=\infty)$ with the computational advantages of linear interpolation $(t=0)$. The computational advantage of a finite t arises from the exponential damping of $N(x, t)$ as compared with the slow damping of $\sin \pi x/\pi x$. (Received October 2, 1944.)

279. I. J. Schoenberg: *On smoothing and subtabulation of empirical functions by means of heat-flow.* II.

Given a sequence of ordinates to be smoothed and subtabulated; the parameters $\{f_n\}$ of the interpolating function $F(x, t)$ of the previous paper are now determined so as to minimize $S=\sum_{n=-\infty}^{\infty}\{F(n, t)-y_n\}^2+\epsilon\sum_{n=-\infty}^{\infty}(f_n-y_n)^2$, where ϵ is a positive smoothing parameter. For $\epsilon=0$ the previous interpolation problem results. For $\epsilon=\infty$, $f_n=y_n$ and $F(x, t)$ is the smoothed version by heat-flow of the polygonal line $F(x, 0)$ of vertices (n, y_n). A compromise between strict interpolation $(\epsilon=0)$ and pure smoothing $(\epsilon=\infty)$ gives the explicit solution (1) $F(x, t, \epsilon)=\sum_{n=-\infty}^{\infty} y_n N(x-n, t, \epsilon)$, where $N(x, t, \epsilon)=(2\pi)^{-1}\int_{-\infty}^{\infty}\{(\epsilon+\phi(u))/(\epsilon+\phi^2(u))\}\psi(u)\cos ux\, du$. Clearly $N(x, t, 0)$ is identical with $N(x, t)$ of the previous paper. Eight-place tables have just been computed on punched card machines for the function $N(x, t, \epsilon)$ and its derivatives $N_x'(x, t, \epsilon)$, $N_x''(x, t, \epsilon)$ for $t=1/2$, $\epsilon=0, 0.1, 0.2, \cdots, 0.9, 1.0$ and the range $-18.5\leqq x\leqq 18.5$ (step 0.1) outside of which these functions vanish to 8 places. On increasing the smoothing parameter ϵ, the approximation (1) becomes smoother in the following sense: If $\sum y_n<\infty$ all integrals $\int_{-\infty}^{\infty}\{F^{(k)}(x, t, \epsilon)\}^2dx$ $(k=0, 1, 2, \cdots)$ exist and each is a monotone decreasing function of ϵ in the range $0\leqq\epsilon<\infty$. This procedure is being applied to certain empirical functions for which very smooth tables are required. (Received October 2, 1944.)

280. Alexander Weinstein and J. R. Pounder: *On two elementary problems of mechanics and electromagnetic theory.*

It is shown that the problem of the motion of a heavy particle on a rotating earth and the problem of the motion of a point charge in uniform electric and magnetic fields are mathematically equivalent, except for a change of axes; whereas they are usually treated by different methods. (Received September 29, 1944.)

Geometry

281. J. E. Wilkins: *A special class of surfaces in projective differential geometry.* II.

In this paper, which is a sequel to one appearing under the same title (Duke Math. J. vol. 10 (1943) pp. 667–675), a more intensive study is made of surfaces in