identity, the radical is defined by using the concept of a quasi-inverse due to Perlis. It is shown that N is a two-sided ideal and it coincides with the left radical, defined in a similar manner. If A has an identity, N is the intersection of the maximal right (left) ideals of A. The results of Stone and of McCoy are a consequence of this theorem. The author has also investigated the radical of an arbitrary algebra and of a normed ring and in the latter case has obtained the criterion: $z\in N$ if and only if $(za)^n\to 0$ for every a in A. (Received August 30, 1944.)

267. Jakob Levitzki: *Chain-conditions and nilpotency.*

Let S be an arbitrary ring. Denote by N_σ, N, and N_γ the sum of all nilpotent, semi-nilpotent and nil-ideals of S, respectively. R. Brauer (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 752–758) by a simple argument proved that if the minimal condition holds for the ideals of S contained in N_σ, then N_σ is nilpotent. By a similar argument the author derived a characteristic condition for the nilpotency of N_σ (Duke Math. J. vol. 11 (1944) pp. 367–368). In the present note, characteristic minimal and maximal conditions are derived for the nilpotency of N_σ, N and N_γ. These results are corollaries of the following theorems: Denote by $R_1, R_2, \cdots$ resp. by $L_1, L_2, \cdots$ infinite sequences of right resp. left-ideals of a nil-subring T of S, then T is nilpotent if and only if: I. Each descending chain of the form $SL_1\supset SL_1L_2\supset\cdots$ and of the form $R_1S\supset R_2R_1S\supset\cdots$ is finite. II. Each ascending chain of the form $(O:L_1)_r\subset(O:L_1L_2)_r$ and of the form $(O:R_1)_l\subset(O:R_2R_1)_l\subset\cdots$ is finite. These theorems yield as a consequence various characteristic conditions for the semi-primarity of a ring. (Received September 7, 1944.)

ANALYSIS

268. R. P. Agnew: *A simple and natural notation for the theory of summability of series and sequences.*

It is proposed that methods of summability be regarded as *operators*, and that the operational (that is, functional) notation be employed in the theory of summability. Thus the statement that a given sequence $s_0, s_1, s_2, \cdots$ or $\{s_n\}$ is summable to σ by a given method A is represented by $\sigma=A\{s_0, s_1, \cdots\}$ or $\sigma=A\{s_n\}$. The statement that a series $u_0+u_1+\cdots$ or $\sum u_n$ is summable B to σ is abbreviated to $\sigma=B\{u_0+u_1+\cdots\}$ or $\sigma=B\{\sum u_n\}$. Discussions and examples are given to illustrate the notation which, the author believes, should have been universally adopted many years ago. (Received September 28, 1944.)

269. R. P. Agnew: *Spans in Lebesgue and uniform spaces of translations of step functions.*

It is shown that, for each $p>1$, the closure in the Lebesgue space L_p of the linear manifold determined by the translations of a given simple step function is the whole space L_p. An explicit formula is given for the approximation of one simple step function by linear combinations of translations of another. (Received September 23, 1944.)

270. E. F. Beckenbach: *Concerning the definition of harmonic functions.*

The following result, which may be compared with the Looman-Menchoff theorem concerning the Cauchy-Riemann first order partial differential equations, is established: If the real function $u(x, y)$ and its first order partial derivatives are continuous in the finite domain D, and if the second order partial derivatives $\partial^2 u/\partial x^2$ and $\partial^2 u/\partial y^2$ exist at all points of D except at most at the points of a denumerable set of points in D, are Lebesgue integrable in D, and satisfy the Laplace equation almost everywhere in D, then $u(x, y)$ is harmonic in D. (Received October 2, 1944.)

271. R. P. Boas and Harry Pollard: *Completeness of sets of functions.* I. *The Legendre functions on* $(-1,1)$. Preliminary report.

The authors discuss the completeness of sets of Legendre functions $P_{\lambda_n}(x)$ for values of the subscripts which are not necessarily integral. By use of Abel's integral equation and Young's inequality it is shown that these functions are complete $L^p(-1,1)$ if the functions $\{\cos \lambda_n x\}$ are complete $L^p(0,\pi)$ and only if $\{\sin \lambda_n x\}$ are complete $L_p(0,\pi)$. Further generalizations are obtained. (Received September 30, 1944.)

272. B. M. Ingersoll: *On singularities of solutions of linear partial differential equations.* III.

It is a well known result for second order linear partial differential equations of hyperbolic type, (1) $u_{xy}+a(x, y)u_x+b(x, y)u_y+c(x, y)u+d(x, y)=0$, that if a, b, c, d are continuous, a unique solution $u(x, y)$ of (1) is determined by the prescription of $u(x, 0)$, $u(0, y)$, if $u(x, 0)$ and $u(0, y)$ are both of class $C^{(1)}$ and $u(x, 0)_{x=0}=u(0, y)_{y=0}$. This paper contains a generalization of this classic boundary value problem. It is shown that if a, b, c, d are of class $C^{(k-1)}$ and if three determinants, the elements of which consist of linear combinations of the coefficients a, b, c, d and their derivatives evaluated at the origin, are not zero, then a unique solution $u(x, y)$ of (1) over a certain rectangular domain is determined by the prescription of $(\partial^k u(x, y)/\partial y^k)_{y=0}=f(x)$ and $(\partial^k u(x, y)/\partial x^k)_{x=0}=g(y)$, where $f(x)$ and $g(y)$ are subject to the conditions that they both be of class $C^{(k)}$ and that $f^{(k)}(0)=g^{(k)}(0)$. (Received September 28, 1944.)

273. R. E. Lane: *The convergence and values of periodic continued fractions.*

Two elementary results from the theory of linear fractional transformations are used to provide a simple, concise proof of a theorem (originally presented by Otto Stolz in a different form) which gives conditions necessary and sufficient for the convergence of the periodic continued fraction. (Received August 13, 1944.)

274. Raphael Salem: *On power series with integral coefficients.*

The paper contains results on the behavior of power series with integral coefficients and their connections with certain classes of algebraic integers. A new proof is given of the closure of the set of algebraic integers whose conjugates lie all inside the unit circle (see Duke Math. J. vol. 11 (1944) p. 103). It is proved that a power series with integral coefficients converging in the unit circle takes values arbitrarily close to any given number, unless it represents a rational function. This theorem is extended in various ways, in particular to monomorphic functions. Also a power series with integral coefficients can not be schlicht in the unit circle unless it is a rational function. The paper gives some properties of the class of algebraic integers having one conjugate inside the unit circle, and all others on the unit circle. The distribution of these numbers is studied, as well as their connection with power series with integral coefficients. (Received September 23, 1944.)

275. Otto Szász: *On Lebesgue summability and its generalization to integrals.*

Hardy and Littlewood proved that $(C, -\alpha)$ summability for some positive $\alpha < 1$ implies Lebesgue summability. The author proves the following generalization: if a series is summable $(C, 1-\alpha)$ and the absolute values of the $(C, -\alpha)$ means are bounded $(C, 1)$, then the series is Lebesgue summable. This summation method is then generalized to integrals and to a general class of trigonometric series. (Received September 29, 1944.)

Applied Mathematics

276. Stefan Bergman: *Representation of potentials of electric charge distributions.*

The author investigates the harmonic functions $V(x_k)$, $k=1, 2, 3$, obtained as potentials of charges of the linear density $F(\zeta)$ along curves $[x_k^* = \xi_k(\zeta),\ k=1, 2, 3,\ \zeta_0 \leqq \zeta \leqq \zeta_1]$, where F and the ξ_k are rational functions of ζ. $V(\zeta_0, \zeta_1; x_k)$ is a hyperelliptic integral, $\int_{\zeta_0}^{\zeta_1} F(\zeta)\left[\prod_\nu(\zeta - a_\nu)\right]^{-1/2} d\zeta$, where the a_ν are algebraic functions of the x_k. Generalizing his previous considerations (see Math. Ann. vol. 101 (1929) p. 534 and Bull. Amer. Math. Soc. vol. 49 (1943) p. 163) and using classical results, the author studies the $V(\zeta_0, \zeta_1; x_k)$ considered as functions of the x_k. Every V can be represented as a sum of the integrals of the first, second and third kind. The periods $\omega_{\alpha\beta}(x_k)$ of the integrals of the first kind are entire functions of the a_ν. The periods of the integrals of the second and third kinds can be expressed in a closed form using θ-functions in terms of the $\omega_{\alpha\beta}(x_k)$. Every $V(\zeta_0, \zeta_1; x_k)$ can be expressed in a closed form using θ-functions in terms of the $\omega_{\alpha\beta}(x_k)$ and the integrals of the first kind. Analogous results are obtained for the potentials V considered as functions of the x_k and some additional parameters Y_n, say the coefficients of the function $\xi_k(\zeta)$. (Received September 29, 1944.)

277. H. W. Eves: *Calculating machine computation forms for the three-point problem on a rectangular coordinate system.*

The three-point problem of surveying has been solved by plane table methods, mechanical methods, trigonometrical methods, geometrical construction methods, and coordinate geometry methods. In view of the general trend to tie surveys into the various state plane coordinate systems, and because of the increasing adoption by engineering concerns of calculating machine procedures to replace the former logarithmic ones, the last, or coordinate geometry methods, are considered the most important. In this paper four different geometrical construction solutions are utilized in building up four corresponding calculating machine forms for finding the coordinates of the observation point in terms of the coordinates of the three observed points and the two observed angles. These forms do not exhibit any very great differences in length or simplicity, running from one of forty entries to one of fifty-six entries, and involving more or less the same combinations of the given elements. One of the forms is essentially that developed and used by the Tennessee Valley Authority. The others are believed to be new. (Received September 11, 1944.)

278. I. J. Schoenberg: *An interpolation formula derived from heat-flow.* I.

Let $\{f_n\}$ $(-\infty < n < \infty)$ be a real sequence. Consider the function $F(x, 0)$ whose graph is the polygonal line of vertices (n, f_n) as a temperature distribution along the