k and has exponent of p of -1 if $p-1$ divides k. This paper generalizes this result, proving that all the Schur derivates $\Delta^m S[n, x^k]$ are p-adically bounded with exponent of p not less than $-2m-1-m/(p-1)$ and hence p-adically convergent. Formulas for $\lim_{n\to\infty} \Delta^m S[n, x^k]$ are given in terms of $\lim_{n\to\infty} S[n, x^{k-m}]$. For positive k, $\lim_{n\to\infty} S[n, x^{2k+1}]=0$ and $\lim_{n\to\infty} S[n, x^{2k}]=(-1)^{k-1} B_k(1-p^{2k-1})$, where B_i is the ith Bernouillian number. The Schur derivates of $\{S[n, f(x)]\}$, where $f(x)=\sum_{i=2}^{\infty} a_i x^i$ and the valuation of $a_i \to 0$ as $i\to\infty$, are p-adically bounded and convergent; moreover $\lim_{n\to\infty} S[n, f(x)]=\sum_{i=2}^{\infty} a_i \lim_{n\to\infty} S[n, x^i]$. (Received June 29, 1944.)

205. Gordon Pall: *Note on factorization in quadratic fields.*

It is proved that if the quadratic integer $x_0+x_1\omega$ is primitive, that is x_0 and x_1 are coprime, then the divisors of $x_0+x_1\omega$ of a given norm are uniquely determined up to a unit factor. Conditions are obtained for the existence of factors of a given norm. It is claimed that the necessity for the introduction of ideals should be based not on the statement that factorization is not unique, but rather that factors do not exist. Thus in the arithmetic of ordinary quaternions, factorization of imprimitive quaternions is not unique, but that of primitive quaternions is both possible and unique; and ideals are in that case unnecessary. (Received July 10, 1944.)

206. R. R. Stoll: *Primitive semigroups.*

Let F denote the class of semigroups S each of whose elements s satisfies an equation of the form $s^n=s^m$ $(n>m)$. A semigroup $S\in F$ is called primitive if for each idempotent $e\in S$ there exists no idempotent $f\neq e$ such that $ef=fe=f$. Examples of such semigroups are (a) semigroups of F which contain only one idempotent and (b) semigroups containing a zero and such that each element is nilpotent (nil semigroups). The following structure theorem is proved for primitive semigroups. A primitive semigroup S contains a unique minimal ideal M with these properties: it is a completely simple semigroup without zero (Rees, Proc. Cambridge Philos. Soc. vol. 36 (1940) pp. 387–400), and the difference semigroup of S modulo M is a nil semigroup. Conversely, a semigroup $S\in F$ with this structure is primitive. (Received July 10, 1944.)

Analysis

207. R. P. Agnew: *Abel transforms of Tauberian series.*

Let $\rho_1=.9680448\cdots$; the constant is Euler's constant plus log log 2 minus $2Ei(-\log 2)$. The following assertion is true when $\rho\geqq\rho_1$ and false when $\rho<\rho_1$. Let $u_0+u_1+\cdots$ be a series satisfying the Tauberian condition $n|u_n|<K$. Let L be the set of limit points of the sequence of partial sums of $\sum u_n$. Let $\sigma(t)=\sum t^k u_k$ be the Abel transform of $\sum u_n$. Let L_A denote the set of limit points of $\sigma(t)$; $z''\in L_A$ if there is a sequence t_n such that $0<t_n<1$, $t_n\to 1$, and $\sigma(t_n)\to z''$. To each $z'\in L$ corresponds a $z''\in L_A$ such that $|z'-z''|\leqq\rho \limsup n|u_n|$. (Received July 19, 1944.)

208. R. P. Agnew: *A genesis for Cesàro methods.*

The family C_r of Cesàro methods of summability, $r\neq 0, -1, -2, \cdots$, is and can be defined as the unique class of methods of summability whose members are simultaneously Nörlund methods and Hurwitz-Silverman-Hausdorff methods. The only methods simultaneously Riesz methods and Hurwitz-Silverman-Hausdorff methods are methods Γ_r closely related to the methods C_r. (Received June 16, 1944.)

209. R. P. Agnew: *Criteria for completeness of orthonormal sets and summability of Fourier series.*

Let $\phi_n(x)$ be an orthonormal set over Euclidean space of one or more dimensions or a measurable subset of such a space. Let G be a convergence-factor method of summability determined by functions $G_n(t)$ defined over a set T having a limit point t_0 not in T and satisfying the conditions (1) $\sum |G_n(t)|^2 < \infty$ for each $t \in T$; (2) $|G_n(t)| < K$ when $t \in T$ and $n = 0, 1, \cdots$; and (3) for each n, $G_n(t) \to 1$ as $t \to t_0$. Several criteria, involving the kernel $K(x, y, t)$ defined as the limit in mean of the partial sums of the series $\sum G_n(t)\phi_n(x)\overline{\phi_n(y)}$, for completeness of the set $\phi_n(x)$ are obtained. It is shown that Fourier series of functions in L_2 are essentially summable by many non-regular as well as regular methods of summability. (Received June 5, 1944.)

210. E. F. Beckenbach and R. H. Bing: *Concerning the vertex mean-value property of harmonic polynomials.*

Results concerning harmonic polynomials and vertex mean-values, by Walsh (Bull. Amer. Math. Soc. vol. 42 (1936) pp. 923–930) for all regular n-gons in the domain of definition, and by Beckenbach and Reade (Trans. Amer. Math. Soc. vol. 53 (1943) pp. 230–238) for oriented regular n-gons, are given under weakened hypotheses. For example it is shown that if $f(x, y)$ is defined in the square S: $0 < x < 1$, $0 < y < 1$, if for each regular n-gon in S the value of $f(x, y)$ at the center of the n-gon is equal to the average of the values of $f(x, y)$ on the vertices, and if there is an M such that the exterior measure of the set of values of (x, y) on which $f(x, y)$ is greater than M is less than 1, then $f(x, y)$ is a harmonic polynomial of degree at most $n-1$. (Received June 26, 1944.)

211. Stefan Bergman: *A class of nonlinear partial differential equations and their properties.*

If one assumes that the thermal conductivity, μ, changes according to the law $\mu = \chi(U)Q(x, y)$ where χ is a function of temperature, U, alone then the heat equation for a steady, two-dimensional heat flow assumes the form $N(U) = U_{z\bar{z}} + aU_z + \bar{a}U_{\bar{z}} + \chi^{-1}\chi_U U_z U_{\bar{z}} = 0$. The function $\Phi(U)$, where $\Phi(U) = c_1\int_0^U \chi dU + c_2$, c_1 and c_2 being constants, satisfies a linear equation $L(\Phi) \equiv \Phi_{z\bar{z}} + a\Phi_z + \bar{a}\Phi_{\bar{z}} = 0$. Complex solutions ϕ of $L(\phi) = 0$, $\Phi = \mathrm{Re}(\phi)$, were introduced in the paper in Trans. Amer. Math. Soc. vol. 53 (1943) pp. 130–155 and other papers cited there, and it was shown that many results of the theory of analytic functions of a complex variable may be generalized to the case of such functions ϕ. Let $U = \Lambda(\Phi)$ be the function inverse to $\Phi = \Phi(U)$. The author introduces complex solutions $u = \Lambda(\phi)$ of $N(\phi) = 0$ and shows that certain theorems of the theory of analytic functions hold (in a conveniently altered form) for the u's. In particular if $\chi = 1/U$ then every u which is "meromorphic" in the domain B can be represented there by $u = [\prod n_B(z, \nu_k)/\prod n_B(z, \mu_k)]u_1$, ν_k and μ_k being the zeros and poles of u. Here u_1, $N(u_1) = 0$, is regular and nonvanishing in B; and n_B, where $N(n_B) = 0$, vanishes only at one point and has boundary values 1 on the boundary of B. (Received June 30, 1944.)

212. R. H. Cameron and W. T. Martin: *Evaluation of various Wiener integrals by use of certain Sturm-Liouville differential equations.*

In this paper the authors evaluate a number of Wiener integrals whose integrands involve exponentials of integrals of the square of the variable function. These evaluations are based on the following theorem. Let $p(t)$ be continuous and positive on $0 \leqq t \leqq 1$, let λ_0 be the least characteristic value of $f''(t)+\lambda p(t)f(t)=0$, subject to the boundary conditions $f(0)=f'(1)=0$, and let $f_\lambda(t)$ denote a non-trivial solution of the above differential equation satisfying $f'(1)=0$. Then if $-\infty<\lambda<\lambda_0$ and $F(x)$ is a functional which makes either side of the following equation exist, one has $\int_C^W F(x) \exp [\lambda \int_0^1 p(t)x^2(t)dt]d_W x = [f_\lambda(1)/f_\lambda(0)]^{1/2}\int_C^W F[y(\cdot) + f_\lambda(\cdot)\int_0^{(\cdot)} f_\lambda'(s)[f_\lambda(s)]^{-2} \cdot y(s)ds]d_W y$. (Received June 3, 1944.)

213. R. H. Cameron and W. T. Martin: *The Wiener measure of Hilbert neighborhoods in the space of real continuous functions.*

Consider the space C consisting of all real functions $x(t)$ continuous on $0 \leqq t \leqq 1$ and vanishing at $t=0$. In an earlier paper (*An expression for the solution of a class of non-linear integral equations*, Amer. J. Math. vol. 46 (1944) pp. 281–298) the authors showed that for every positive number R the subset of C for which $\int_0^1 [x^2(t)]^2 dt < R^2$ has positive Wiener measure. In the present paper the authors evaluate the measure of this set and in addition evaluate Wiener integrals of functionals $F[\int_0^1 x'(t)dt]$ over the space C for a general class of functions $F(u)$. The evaluation is in terms of the theta-function of the first kind. Integrals are also evaluated for the case of functions $x(t)$ not necessarily vanishing at $t=0$. (Received June 2, 1944.)

214. J. J. Dennis: *Some points in the theory of positive definite J-fractions.*

Wall and Wetzel ([1] Trans. Amer. Math. Soc. vol. 55 (1944) pp. 373–392; [2] Duke Math. J. vol. 11 (1944) pp. 89–102) have extended a considerable part of the Stieltjes theory to "positive definite J-fractions" (1) $1/(b_1+z)-a_1^2/(b_2+z)-a_2^2/(b_3+z)- \cdots$, characterized by the condition that the quadratic forms (2) $\sum_{p=1}^{n}(I(b_p)+y)x_p^2-2\sum_{p=1}^{n-1}I(a_p)x_p x_{p+1}$ are positive definite for $y>0$. In [1] they build upon the determinant inequalities $D_n(y)>0$, where $D_n(y)$ is the discriminant of (2). In [2] they formulate the condition of positive definiteness without determinants as (3) $I(b_p)\supseteqq 0, |a_p^2| - R(a_p^2) \leqq 2I(b_p)I(b_{p+1})(1-g_{p-1})g_p, 0 \leqq g_{p-1} \leqq 1, p=1, 2, 3, \cdots$, and are able to connect the theory with other work on continued fractions. The present paper is concerned with the problem: To simplify the theory in [1] by building upon the inequalities (3). The formulation (3) is proved without determinants; and an extension of a certain minimum property of (2) [2, §3] is given. The "nest of circles" is obtained and their properties established in a very simple way. Extensions of certain theorems of [2] are given, for instance, a "best" extension of Szász' theorem. Certain parts of the theory are carried over to (1′) $1/(b_1+z_1)-a_1^2/(b_2+z_2)-a_2^2/(b_3+z_3)- \cdots$. (Received July 5, 1944.)

215. Nelson Dunford and Einar Hille: *The differentiability and uniqueness of continuous solutions of addition formulas.*

Let $G(u, v)$ be a single-valued analytic function of u, v. If $f(\lambda)$ is continuous for $0 \leqq \lambda \leqq \alpha$ and has its values in a commutative normed ring and satisfies the equation $f(\lambda+\mu)=G(f(\lambda), f(\mu))$ and if $G_1(f(0), f(0))$, where $G_1=\partial G/\partial u$, has an inverse then $f(\lambda)$ has derivatives of all orders and any such solution is determined by $f(0), f'(0)$. Analogous questions are discussed in the case that λ, μ are elements of a normed ring and also in the case where f has its values in an operator ring and is continuous in the strong topology. (Received July 8, 1944.)

216. Paul Erdös: *Converse of Fabry's gap theorem.*

Let $n_1 < n_2 < \cdots < n_k < \cdots$ be a sequence of integers with $\lim (n_k/k) = \infty$. The gap theorem of Fabry states that if $\sum a_k z^{n_k}$ is a power series whose circle of convergence is the unit circle then the unit circle is the natural boundary. Pólya proved the following converse of this theorem: Let $n_1 < n_2 < \cdots < n_k < \cdots$ be a sequence such that $\liminf (n_k/k) < \infty$, then there exists a power series $\sum a_k z^{n_k}$ whose circle of convergence is the unit circle and the unit circle is not the natural boundary. For this theorem of Pólya a simple and elementary proof has been obtained. (Received August 1, 1944.)

217. B. M. Ingersoll: *On singularities of solutions of linear partial differential equations.* II.

In continuation of his previous investigation (see abstract 50-3-60), the author considers the problem of locating singularities of real solutions $u = \sum_{m,n=0}^{\infty} D_{mn} z^m \bar{z}^n$, $D_{mn} = \overline{D}_{nm}$, and of complex solutions $u = \sum_{m,n=0}^{\infty} A_{mn} z^m \bar{z}^n$, $z = x + iy$, $\bar{z} = x - iy$, of the equation $L(U) = \Delta U/4 + A^{(1)} U_x + A^{(2)} U_y + A^{(3)} U = 0$ where $A^{(k)} = \sum_{m,n=0}^{\infty} A_{mn}^{(k)} x^m y^n$ are entire functions of x and y. Using a result of Hadamard he determines the location of singularities of U in terms of a subsequence $\{D_{mk}\}$, k fixed, $m = 0, 1, 2, \cdots$ and a finite number of derivatives of $A^{(k)}$, $k = 1, 2, 3$. Further, employing an integral representation of Bergman for the solutions he obtains a majorant $X(z, \bar{z})$ of $|U(z, \bar{z})|$, $U(z, \bar{z}) = \text{Re}[u(z, \bar{z})]$. X depends on a sequence $\{D_{mk}\}$, k fixed, $m = 0, 1, 2, \cdots$, and the upper bounds of the coefficients $A^{(k)}$ of L in any finite region of the plane. He also investigates the character of certain types of singularities of U and the rate of growth of U in the neighborhood of these points. (Received July 10, 1944.)

218. W. H. Ingram: *On the integral equations of continuous dynamical systems.*

The dynamical systems are the one-dimensional ones considered recently (Philosophical Magazine, 1940, Bull. Amer. Math. Soc. vol. 48) and are governed by the equation $\gamma(x, t) : = \int_a^b \mathcal{K}(x, \xi) \dot{\gamma}(\xi, t) : d\xi$. The substitution $\gamma_i(x, t) = W_i(x) T_i(t)$ leads to the equation $\mathcal{B}\dot{T} : = T :$ having the solution $T : = e^{\mu t} C :$ with $C :$ and μ satisfying the *subsidiary modal equation* $\mu \mathcal{B} C : = C :$. The elements of the $n \times n$ matrix $\mathcal{B}$ are obtained by an iterative process. An expansion for $\mathcal{K}(x, y)$ is assumed in establishing convergence. Evidence is cited for the existence of a wide but undefined class of kernels the members of which, or their iterates, may be represented by this expansion. The method for the polynomial approximation of modal functions explained in the Philosophical Magazine and somewhat more general formulas for the algebraic determination of the modal numbers are shown to be valid in a sub-class of practical importance. If $\Phi(x) :$ is any modal vector then $\mathcal{N}\Phi(x) :$ is also with $\mathcal{N}$ the diagonal form of any of the solutions of $\mu \mathcal{B} N : = N :$ and with the corresponding μ of this equation the modal number belonging to $\mathcal{N}\Phi(x) :$. It is found finally that the abstract theory of Hilbert space has no contact with the differential equations of dissipative elastokinetics and therefore little if any application in the present or any related problem. (Received June 6, 1944.)

219. R. L. Jeffery and D. S. Miller: *Convergence factors for generalized integrals.*

In a generalization of the derivative, J. C. Burkill (Proc. London Math. Soc. (2) vol. 34 (1932) pp. 314–322) used the Cesàro mean of a function $F(x)$ defined by

$C_r(F, x, x+h)=(r/h^r)\int_x^{x+h}(x+h-t)F(t)dt$. For the first mean of an interval (a, b), $C_1(F, a, b)=(b-a)^{-1}\int_a^b F(t)dt=(a-b)^{-1}\int_b^a F(t)dt=C_1(F, b, a)$. For $r>1$, $C_r(F, a, b)$ is in general not equal to $C_r(F, b, a)$. Consequently, the methods used for inverting derivatives of the first order could not be used for derivatives of a higher order, and this led to the following consideration. The expression $(x+h-t)^{r-1}$ serves as a convergence factor in $C_r(F, x, x+h)$. Would it be possible to replace this by a convergence factor defined in such a way that means corresponding to $C_r(F, a, b)$ and $C_r(F, b, a)$ are equal? The present paper is an answer to this question. It turns out that there is a wide class of suitable factors. The results parallel those of Burkill, but owing to the general form of the convergence factors, it was necessary to devise new methods of proof. (Received July 24, 1944.)

220. D. S. Miller: *Some properties of Carathéodory linearly measurable plane point sets.*

In 1920 H. Steinhaus (see Fund. Math. vol. 1 (1920) pp. 93–104) proved that the set of distances between the points of a measurable set along the line of positive measure fills up an interval about the origin. An application of Fubini's theorem at once establishes the same result in the plane when ordinary Lebesgue measure is used. In this paper plane sets measurable in the sense of Carathéodory are considered. It will be shown that nonzero measurability in the Carathéodory sense is not enough to assure that the set of distances between the points fills up an interval. Also some consideration will be given to the Δ-density function first invented by Besicovitch (see Math. Ann. vol. 98 (1927) pp. 422–464). It will be shown that $3^{1/2}/2 \leqq \overline{\Delta}(A, p) \leqq 1$ for almost all points of the set A. Furthermore it will turn out that this is the best possible relation. (Received June 20, 1944.)

221. C. N. Moore: *Convergence factors in general analysis*, I.

In several previous papers (Proc. Nat. Acad. Sci. U.S.A. vol. 8 (1922) pp. 288–293, Trans. Amer. Math. Soc. vol. 24 (1922) pp. 79–88, vol. 25 (1923) pp. 459–468) the author has developed the basis of a branch of general analysis which would include the results common to the theory of infinite series and the theory of infinite integrals. In these papers the usefulness of the general theory was illustrated by proving a theorem in it which included as special cases the Knopp-Schnee-Ford theorem with regard to the equivalence of the Cesàro and Hölder means in summing divergent series, the analogous theorem of Landau concerning summable integrals, and a further new theorem concerning the equivalence of generalized derivatives of the Cesàro and Hölder type. In the present paper certain theorems are proved concerning the introduction of convergence factors into operations of the type $(J\eta)(\sigma)$, defined in the papers referred to above. These theorems include theorems concerning infinite series in vol. 22 of the Society's Colloquium Publications and analogous theorems concerning infinite integrals. Further theorems of the type indicated will be considered in later papers. (Received July 5, 1944.)

222. E. N. Nilson and J. L. Walsh: *On functions analytic in a region: approximation in the sense of least pth powers.*

Let R be a finite region bounded by a finite sum of Jordan curves C_1, and let S be a closed set interior to R with boundary C_0 a finite sum of Jordan curves which separates no point of $R-S$ from C_1. Let $f(z)$ be a function analytic on S but not throughout R. Let $\mathfrak{F}_M(z)$ be a function analytic in R such that the integral mean of

$|\mathfrak{F}_M(z)|^q$ $(q>0)$ on C_1, with respect to the conjugate of the harmonic function equal to zero and unity on C_0 resp. C_1, is not greater than M and such that the integral mean of $|f(z)-\mathfrak{F}_M(z)|^p$ $(p>0)$ on C_0 is least. By means of a generalization of the Hardy Mean-Value Theorem, the authors consider the asymptotic behavior of $\{\mathfrak{F}_M(z)\}$ as $M\to\infty$, extending the results of their earlier paper (Trans. Amer. Math Soc. vol. 55 (1944) pp. 53–67). These functions of best approximation are characterized for the special case of circular regions and least squares. (Received June 5, 1944.)

223. Harry Pollard: *One-sided boundedness as a condition for the unique solution of certain heat equations.*

The mathematical problem (1) $u_{xx}=u_t$ $(-\infty<x<\infty, 0<t<c)$, (2) $u(x, 0+)=f(x)$, for heat flow in an infinite rod with prescribed initial temperature, is not uniquely determined. For if $u(x, t)$ is a solution, so is $u+xt^{-3/2}\exp(-x^2/4t)$. Various additional conditions on $|u(x, t)|$ which guarantee uniqueness are known. A new, one-sided condition is suggested by the phenomenon of absolute zero. Mathematically this may be stated as: (3) $u(x, t)\geqq -M$. It is proved in this paper that solution of problem (1)-(2)-(3), if it exists, is unique. For $f(x)\equiv 0$ and $M=0$ this result is due to Widder. A result of similar character is obtained for functions harmonic in a half-plane. (Received July 6, 1944).

224. P. C. Rosenbloom: *Interpolation and extremal problems for absolutely monotonic functions.*

Let F be the class of functions $f(x)=\sum_{n=0}^{\infty}a_nx^n$ which are absolutely monotonic in the interval $0\leqq x\leqq 1$ and satisfy the conditions $f(x_i)=m_i$, $i=1, \cdots, k-1$, $f(1)\leqq m_k$, where $-1<x_1<\cdots<x_{k-1}<1$, and $x_1, \cdots, x_{k-1}$, $m_1, \cdots, m_k$ are given numbers. Let $L(f)=\sum_{n=0}^{\infty}c_na_n$, where $c_n\to 0$. Then $L(f)$ attains its maximum in the class F for a polynomial with at most k nonvanishing coefficients. A necessary and sufficient condition that F be non-empty is that it contain such a polynomial. As an application, the following problem is solved. Let $g(z)$ be analytic in $|z|<r_3$, and let its mean square be prescribed on the circles $|z|=r_1$ and $|z|=r_3$, $0<r_1<r_3$. For a given circle $|z|=r_2$, $r_1<r_2<r_3$, what is the greatest possible value of the mean square of $g(z)$? The analogous problem for the maximum modulus was studied by Heins in a recent paper, Trans. Amer. Math. Soc. vol. 55 (1944). Banach space methods are used in the proofs. (Received July 7, 1944.)

225. Otto Szász: *On the generalized jump of a function and Gibbs' phenomenon.*

An odd function $f(t)$ is said to have a generalized jump of index r at the origin if its Cesàro mean of order r has a limit not equal to 0, as t decreases to zero. In this paper, generalizing earlier results, formulae are given to determine the jump by means of the Fourier coefficients, when $r<3$. Furthermore in this case a Gibbs' phenomenon is established. Certain trigonometric transforms and their relation to Cesàro summability are employed. (Received June 28, 1944.)

226. W. J. Trjitzinsky: *Singular elliptic and hyperbolic partial differential equations.*

In this paper the author studies the elliptic equation $F(u)=f(x)$, where (E) $F(u)\equiv\sum_i\sum_j\partial(A_{ij}(x)(\partial u/\partial x_i))/\partial x_j+C(x)u$ $(i, j=1, \cdots, m;\ x=(x_1, \cdots, x_m))$ and the related hyperbolic equation of normal type (H) $F(u)=\rho(x)\partial^2u/\partial t^2+\rho(x)f(x, t)$

($\rho(x)>0$; $-\infty<t\leqq t_0$). The coefficients are continuous and suitably differentiable in $x_1, \cdots, x_m$ for x in a bounded open domain D; they are allowed to become infinite in the neighborhood of the frontier $F(D)$ of D; $F(D)$ may be irregular. (E), (H) are transformed into an integral equation, whose kernel $K(x, z)$ is fundamental for our theory. When $K(x, z)$ is L_2 in a certain one of the variables, while $F(u)$ is self-adjoint, (E), (H) can be effectively studied by methods of spectral theory; when $K(x, z)$ is L_2 in (x, z), Fredholm's theory of integral equations is applicable even when $F(u)$ is not self-adjoint. In the first case it is said the problems are of type (S); in the second—of type (F). Explicit conditions on the coefficients in (E), (H) are found under which the problems are of types (S) or (F). The spectral theory then is developed, yielding various results on existence of solutions, their properties, conditions for their uniqueness, and so on. This work is related to the author's and T. Carleman's earlier works on elliptic partial differential equations. (Received July 8, 1944.)

227. H. S. Wall: *Note on the expansion of a power series into a continued fraction.*

This paper contains an algorithm for expanding a power series into a continued fraction which is based upon the fact that the process for constructing a sequence of orthogonal polynomials can be so arranged that it gives simultaneously a continued fraction expansion for a power series. (Received June 5, 1944.)

228. H. S. Wall: *The convergence of a positive definite J-fraction in the limit-circle case.*

Theorems 4.1, 4.2, 4.3 and 4.4 of the paper *Contributions to the analytic theory of continued fractions and infinite matrices* by E. D. Hellinger and H. S. Wall, Ann. of Math. vol. 44 (1943) pp. 103–127, are extended, with appropriate modifications in the series (4.7), (4.8) and in the polynomials (4.12), to general positive definite J-fractions. Thus, in the limit-circle case, a positive definite J-fraction must either converge over the whole plane to a meromorphic limit-function, or else diverge for every value of the variable. (Received July 13, 1944.)

Applied Mathematics

229. Nathaniel Coburn: *The Kármán-Tsien pressure-volume relation in the two-dimensional supersonic flow of compressible fluids.*

First, the applicability of the Kármán-Tsien idea in the supersonic range is discussed. Secondly, it is shown that when the Kármán-Tsien relation can be used, the characteristics form a Tschebyscheff net. Further, if the diagonal curves of these characteristics are drawn so as to correspond to equi-intervaled values of the arc length parameter along these characteristics, then these diagonal curves will be the families of equipotentials and stream lines. Analytically, this last result means that the determination of the stream lines depends upon two arbitrary functions of a real variable. The conditions satisfied by these functions are discussed in the case where the given data is of Dirichlet type (two known stream lines as in the jet problem). In particular, if one of the known stream lines coincides with the x-axis, it is shown that only one arbitrary function enters into the problem of determining the stream lines. (Received June 2, 1944.)