

known recursion relationships, and checking by a neat formula given in Jordan's *Calculus of finite differences*) and then multiplying by $m!/s!$. The numbers $B_{m,s}$ are expressed in lowest terms. (Received April 20, 1944.)

190. Seymour Sherman, J. DiPaola, and H. F. Frissel: *Routh's discriminant, flutter, and ground resonance*. Preliminary report.

Routh's criterion for the stability of the solutions of system of linear differential equations with constant coefficients is extended to cover cases arising in airplane flutter and helicopter ground resonance calculations. With this new tool, the stability of the flutter "polynomial" at a given reduced frequency for more than two degrees of freedom can be determined in one-fifth of the time hitherto required. (Received May 29, 1944.)

GEOMETRY

191. Edward Kasner and John DeCicco: *Isothermal families in general curvilinear coordinates, and loxodromes*.

If the square of the linear element of a surface Σ is given in isothermal coordinates (x, y) by $ds^2 = E(x, y)(dx^2 + dy^2)$, then the family of curves $g(x, y) = \text{const.}$ on Σ is isothermal if and only if $(\partial^2/\partial x^2 + \partial^2/\partial y^2) \arctan g_x/g_y = 0$. In the present paper, the authors obtain the necessary and sufficient condition that $g(x, y) = \text{const.}$ represent an isothermal family when (x, y) are general curvilinear coordinates. This gives a large extension of Lie's theorem. The condition is simpler when the parametric curves form an orthogonal net. As an application, the condition is obtained that $g(x, y) = \text{const.}$ represent an isothermal family upon the Cartesian surface $z = f(x, y)$. Finally the condition is found that the level curves of the surface be an isothermal family. This is applied to the mapping of loxodromes, showing that they can be represented by straight lines for a sphere (Mercator) and spheroid (Lambert), but not for an ellipsoid of three unequal axes. Use is made of Kasner's theorem in *Math. Ann.* (1904). (Received April 20, 1944.)

192. Abraham Seidenberg: *Valuation ideals in polynomial rings*.

A constructive study of the valuation ideals in a polynomial ring $\mathfrak{D} = K[x, y]$ in two indeterminates, where K is an algebraically closed (ground-) field, is made. Let $\mathfrak{q}_1, \mathfrak{q}_2, \dots$ be the Jordan sequence of v -ideals belonging to a valuation B of \mathfrak{D}/K , where Σ is the quotient field of \mathfrak{D} , and let \mathfrak{q}_i be the j th ideal such that $v(\mathfrak{q}_i)$ is not in the additive group generated by $v(\mathfrak{q}_1), \dots, v(\mathfrak{q}_{i-1})$. A tool corresponding to the Puiseux series expansion for a valuation, which is available if K is of characteristic 0 but not in general, is found in introducing certain polynomials f_i such that $v(f_i) = v(\mathfrak{q}_i)$. Considerations are reduced to valuations of rational rank 2. If B is of rational rank 2, place $v(\mathfrak{q}_1) = 1$ and let τ be the least irrational value assumed by elements of \mathfrak{D} . The description of the v -ideals in \mathfrak{D} for B is intimately connected with the approximants and quasiapproximants to a certain integral multiple of τ . In particular, a simple 0-dimensional v -ideal \mathfrak{q}_i is characterized in terms of the values of \mathfrak{q}_i and \mathfrak{q}_{i+1} . This characterization yields a proof that the transform of a simple v -ideal under a quadratic transformation is simple. If \mathfrak{q}_i is not simple, an explicit factorization of \mathfrak{q}_i in terms of the mentioned values is given. (Received May 22, 1944.)

193. A. H. WHEELER: *One-sided polyhedra from the five regular solids*.

If planes are passed through a polyhedron in such a way as to permit removing parts and leaving cells which are joined along their edges, and adjoining cells are bounded by one or more planes which are common to the two cells, then one-sided polyhedra can be derived. In this paper the method is applied to the five regular solids and several forms are shown constructed of plastic material. The intersecting planes may be so disposed as to cut away the vertices, the edges in whole or in part, and parts of the faces of the original solids. In particular, in the case of the regular icosahedron, a part of a regular dodecahedron of the second species may be removed from the interior of the solid leaving a group of cells distributed along the edges of the original solid, and the cells can be entered or traversed in more than one way. Two types of one-sided octahedra are shown. (Received May 26, 1944.)

194. Oscar Zariski: *The theorem of Bertini concerning the variable singular points of a linear system of varieties.*

If the ground field k of an algebraic irreducible r -dimensional variety V/k is extended by the adjunction of indeterminates u_1, u_2, \dots, u_m , we denote by V/K the extended variety over $K = k(u)$ which has the same general point (ξ) as V/k . Each subvariety W/k of V/k has similarly a unique extension W/K on V/K , and each subvariety W^*/K of V/K has a unique contraction W/k on V/k . Given $m+1$ linearly independent polynomials $f_i(\xi)$, $i=0, 1, \dots, m$, there is a unique $(r-1)$ -dimensional irreducible subvariety F^*/K of V/K whose general point (η) satisfies the equation $f_0(\eta) + u_1 f_1(\eta) + \dots + u_m f_m(\eta) = 0$. The main result is as follows: (1) W/k is a base variety of the linear system $|F|$ defined on V/k by $f_0 + \lambda_1 f_1 + \dots + \lambda_m f_m = 0$, if and only if W/K is on F^*/K ; (2) if W^*/K is a singular subvariety of F^*/K , then the contraction W/k is either singular for V/k or is a base variety of $|F|$. The theorem of Bertini in its classical formulation ("the variable singular points of $|F|$ are either singular for V/k , or lie on the base locus of $|F|$ ") is equivalent to (2), if k is of characteristic zero, and is false if k is of characteristic $p \neq 0$. (Received April 6, 1944.)

TOPOLOGY

195. W. H. GOTTSCHALK: *Orbit-closure decompositions and almost periodic properties.*

Let X be a metric space with metric ρ , let $f(X) \subset X$ be a continuous mapping, and let $h(X) = X$ be a homeomorphism. For $x \in X$, the set $\sum_{n=0}^{+\infty} f^n(x)$ is called the *semi-orbit* of x under f and the set $n \sum_{-\infty}^{+\infty} h^n(x)$ is called the *orbit* of x under h . The mapping f is said to be *pointwise almost periodic* provided that if $x \in X$, then to each $\epsilon > 0$ there corresponds a positive integer N with the property that in every set of N consecutive positive integers appears an integer n so that $\rho(x, f^n(x)) < \epsilon$. The mapping f is said to be *uniformly pointwise almost periodic* provided that to each $\epsilon > 0$ there corresponds a positive integer N such that if $x \in X$, then in every set of N consecutive positive integers appears an integer n so that $\rho(x, f^n(x)) < \epsilon$. The following theorems are proved: In order that the mapping f (homeomorphism h) give a semi-orbit-closure (orbit-closure) decomposition of X it is sufficient that f (h) be pointwise almost periodic; and in case X is locally compact (compact), this condition is also necessary. In order that the mapping f (homeomorphism h) give a continuous semi-orbit-closure (orbit-closure) decomposition of X it is sufficient that f (h) be uniformly pointwise almost periodic; and in case X is compact, this condition is also necessary. (Received May 15, 1944.)