

to any conformal map of a sphere upon a plane lead to new characterizations of the Mercator, stereographic, and the general Lambert conical projections. Thus the only conformal map with straight scale curves is the Mercator; and the only circular cases are the stereographic and Lambert maps. (Received January 25, 1944.)

STATISTICS AND PROBABILITY

96. Benjamin Epstein and C. W. Churchman: *On the statistics of sensitivity data.*

"Sensitivity data" is a general term for that type of experimental data for which the measurement at any point in the scale destroys the sample. The paper is a generalization of a method of treating such data due to Spearman. (C. Spearman, *The method of "right and wrong cases" (constant stimuli) without Gauss' formulae*, British Journal of Psychology vol. 2 (1908) pp. 227-242.) Formulae for the moments and their standard sampling errors are given. Certain minimization problems are also discussed. (Received January 26, 1944.)

97. E. J. Gumbel: *The observed return period.*

The theoretical return period $T(x)$ of a value equal to, or greater than, x is defined as the inverse of the probability $1 - F(x)$. The question is how to calculate, for n observations, the return period $T(x_m)$ of the m th observation x_m ($m = 1, 2, \dots, n$), and especially $T(x_n)$ of the largest observation x_n for an unlimited variate. This problem is important in probability papers where the variate is plotted as a function of the return period. Engineers use a compromise between the exceedance interval $'T(x_m) = n/(n-m)$ and the recurrence interval $''T(x_m) = n/(n-m+1)$, namely $T(x_m) = n/(n-m+1/2)$. In this case $T(x_n) = 2n$. If, however, the probability $F(\tilde{x}_n)$ of the median \tilde{x}_n of the largest value is attributed to x_n , $T(x_n) = 1.44n + 1/2$. Both methods can hardly be justified. The author attributes the probability $F(\tilde{x}_n)$ of the most probable largest value \tilde{x}_n to x_n . Then $T(x_n)$, as is to be expected, converges toward n , and equals n for the exponential distribution, and $n+1$ for the logistic distribution. In the same way, the probability $F(\tilde{x}_1)$ of the most probable smallest value \tilde{x}_1 is used, for an unlimited variate, as frequency of the smallest observation x_1 . The frequencies $F(\tilde{x}_m)$ of the intermediate $n-2$ observations are obtained by linear interpolation between $F(\tilde{x}_1)$ and $F(\tilde{x}_n)$. Thus the return periods may be determined for all observations. (Received January 27, 1944.)

98. H. E. Robbins: *On the measure of a random set.*

Let X , a measurable subset of Euclidean n -dimensional space E , be a random variable (for example, X may be the set-theoretical sum of N possibly overlapping and independently chosen unit intervals on a line with a given probability distribution for their centers). Let $m(X)$ denote the measure of X , and for any point x of E let $p(x)$ denote the probability that X contain x . Then under very general hypotheses on X it is shown that the expected value of $m(X)$ is equal to the integral over E of $p(x)$. More generally, the expected value of the r th power of $m(X)$ is equal to the integral over r -dimensional space of the function $p(x_1, \dots, x_r)$ = probability that X contain all the points x_1, \dots, x_r . (Received January 28, 1944.)