87. H. E. Salzer: *Table of coefficients for inverse interpolation with advancing differences.*

This table can be used in place of the similar one described in a previous abstract (49-9-224). In addition it has the advantage that it can be employed for inverse interpolation near the beginning or end of a table and also when only a few tabulated values are available (as is the case, for instance, when solving transcendental equations). The Mathematical Tables Project has computed the coefficients of the products of ratios of advancing differences of various order. These coefficients occur in the formula obtained by the inversion of the Gregory-Newton formula for direct interpolation, employing Lagrange's theorem. The polynomial expressions for those coefficients are given in H. T. Davis, *Tables of the higher mathematical functions*, vol. 1, pp. 80–81. (A slight addition to the formula was made to complete it as far as the eighth order.) The coefficients of the two fourth order and the two fifth order terms were calculated to ten decimals, at intervals of 0.001 of the argument $m=(u-u_0)/(u_1-u_0)$. The coefficients of the four sixth order terms were calculated at intervals of 0.01 and the four seventh order coefficients as well as the seven eighth order coefficients were computed at intervals of 0.1 (all to ten decimals). (Received December 2, 1943.)

88. Andrew Vazsonyi: *On two-dimensional rotational gasflows.*

The differential equation of an inviscous compressible fluid is determined under the condition that the conductivity of the gas is negligible. (By admitting discontinuities this includes flows with shock waves.) The equation of motion is (1) $\psi_{xx}(1-(u^2/a^2)) -(2uv/a^2)\psi_{xy}+\psi_{yy}(1-(v^2/a^2))=\rho^2[\partial h_0/\partial\psi-((k-1)/kR)(h_0+q^2/2)\partial s/\partial\psi]$, $u=(1/\rho)\psi_y$, $v=-(1/\rho)\psi_x$, $\rho=e^{-s/R}(h_0-q^2/2)^{1/k-1}$, $q^2(h_0-q^2/2)^{2/k-1}=e^{2s/R}(\psi_x^2+\psi_y^2)$, $a^2=ke^{-(k-1)^{s/R}} \cdot(h_0-q^2/2)$ where the notations are as follows: ψ streamfunction, q velocity, u and v velocity components, ρ density, a local speed of sound, h_0 stagnation enthalpy (Bernoulli constant), s specific entropy, R gas constant, k isentropic exponent. There are two arbitrary functions in this equation, namely: $h_0(\psi)$ and $s(\psi)$; these must be given by the boundary conditions (or by the nature of the discontinuities). The flow is rotational in general and the rotation is given by $\omega=\partial v/\partial x-\partial u/\partial y=-\rho\partial h_0/\partial\psi +(p/R)\partial s/\partial\psi$. For irrotational flow the right-hand side of (1) equals 0. (Received January 28, 1944.)

GEOMETRY

89. Stefan Bergman: *A Hermitian metric and its property.* Preliminary report.

Let the real analytic function $\Phi(z_1, z_2, \bar{z}_1, \bar{z}_2)$, $z_k=x_k+iy_k$, $\bar{z}_k=x_k-iy_k$, $k=1, 2$, of four real variables x_1, x_2, y_1, y_2, satisfy the equation $\Phi=c\{[\partial^2\Psi/\partial z_1\partial\bar{z}_1][\partial^2\Psi/\partial z_2\partial\bar{z}_2] -|\partial^2\Psi/\partial z_1\partial\bar{z}_2|^2\}$, $c=$constant, $\Psi\equiv\log\Phi$, in a domain B of the (four-dimensional) space and become infinite on the boundary of B. The expression $ds_B^2(z_1, z_2) =\sum_{m,n=1}^{2}[\partial^2\Psi/\partial z_m\partial\bar{z}_n]dz_m d\bar{z}_n$ defines in B a Hermitian metric which is invariant with respect to transformations by pairs of analytic functions, $z_k^*=z_k^*(z_1, z_2)$, $k=1, 2$, of two complex variables which are regular in B. Using the methods of the theory of orthogonal functions (see Bergman, *Sur les fonctions orthogonales de plusiers variables complex avec les applications à la théorie des fonctions analytiques*, Interscience Publishers, New York, 1941, chap. 4) the author investigates the property of this metric in the large. The author introduces the system of orthogonal functions, $\{M^1(z_1, z_2), M^{01}(z_1, z_2), \cdots\}$ (see loc. cit. p. 28) which he obtains by replacing in the formula (9) (see loc. cit. p. 46) the kernel function K by Φ. Various applications to the theory of pseudo-conformal transformations are discussed. (Received December 15, 1943.)

90. S. S. Chern: *A simple intrinsic proof of the Gauss-Bonnet formula for Riemannian polyhedra.*

In an n-dimensional Riemannian space let R_{ijkl} be the Riemann-Christoffel tensor. Let $\epsilon^{i_1 \cdots i_n}$ equal 1 or -1, according as the indices form an even or odd permutation of $1, \cdots, n$, and be otherwise zero. Define $S = \epsilon^{i_1 \cdots i_n} \epsilon^{j_1 \cdots j_n} R_{i_1 i_2 j_1 j_2} \cdots R_{i_{n-1} i_n j_{n-1} j_n}$ when n is even and $S=0$ when n is odd. Let the Riemannian space be orientable and closed. The Gauss-Bonnet formula in its simplest form asserts that the integral of S over the volume element is equal to a constant multiple of the Euler characteristic of the space. (Cf. C. B. Allendoerfer and André Weil, Trans. Amer. Math. Soc. vol. 53 pp. 101–129.) A simple intrinsic proof of this formula is given, based on the following two facts: (a) the integrand in question is a null form in the $(2n-1)$-dimensional space formed by the unit vectors of the Riemannian space; (b) a field of unit vectors can be defined in the space with a finite number of singular points. Of these two facts (a) is proved by an explicit formula, while (b) is well known. An application of Stokes' formula in the space of unit vectors immediately yields the desired result. Incidentally the index theorem on vector fields follows as a consequence of the present proof. Suitable modifications give the formula for a Riemannian polyhedron. Under reasonable assumptions the formula holds for an infinite Riemannian space. (Received December 13, 1943.)

91. J. M. Feld: *On a representation in space of groups of circle and turbine transformations in the plane.*

In a previous paper (Bull. Amer. Math. Soc. vol. 48 (1942) pp. 783–790) the author showed that the oriented lineal elements of the euclidean plane can be mapped continuously and 1-1 upon the points of quasi-elliptic three-space, Q_3, so that the group of whirl-similitudes in the plane is isomorphic with the group of automorphic projectivities of Q_3. In this paper the mapping is extended to a 1-1 continuous representation of the lineal elements of the Moebius plane upon the points of projective S_3. The Laguerre, Moebius, and Lie groups of circle transformations are then shown to be isomorphic with groups of projective transformations in S_3 which leave certain linear complexes and congruences of lines invariant, and Kasner's group of turbine transformations is shown to be isomorphic with the projective group of S_3. (Received January 29, 1944.)

92. C. C. Hsiung: *Some invariants of certain pairs of hypersurfaces.*

The purpose of this paper is to generalize the results of Buzano (Bollettino della Unione Matematica Italiana vol. 14 (1935) pp. 93–98), Bompiani (Bollettino della Unione Matematica Italiana vol. 14 (1935) pp. 237–243), and the author (*Projective invariants of a pair of surfaces*, to appear in Amer. J. Math.). Two pairs of hypersurfaces V_{n-1}, V^*_{n-1} in a space S_n of n ($\geqq 3$) dimensions are considered: (a) The tangent hyperplanes t_{n-1}, t^*_{n-1} at two ordinary points O, O^* are coincident. (b) The tangent hyperplanes t_{n-1}, t^*_{n-1} at two ordinary points O, O^* are distinct, and the common tangent flat space t_{n-2} of t_{n-1}, t^*_{n-1} contains the line OO^*. It is shown that determined by the neighborhoods of the second order of the hypersurfaces V_{n-1}, V^*_{n-1} at the points O, O^* there exist a unique and two projective invariants for the two cases respectively; and each invariant is given metrically as well as projectively geometric characterizations. (Received December 29, 1943.)

93. G. B. Huff: *The completion of a theorem of Kantor.*

Let a complete and regular linear system Σ be defined by its order x_0 and its multiplicities $x_1, \cdots, x_\rho$ at a set, P^2_ρ, of ρ general points in the plane. A planar Cremona transformation C whose fundamental points fall at P^2_ρ will send Σ into a system Σ' of order x'_0 and multiplicities $x'_1, \cdots, x'_\rho$ at the fundamental points of C^{-1}. The characteristic $\{x\} = \{x_0; x_1, \cdots, x_\rho\}$ of Σ is related to $\{x'\} = \{x'_0; x'_1, \cdots, x'_\rho\}$ by a linear transformation L which has integral coefficients and the invariant forms $(xx) \equiv x_1^2 + \cdots + x_\rho^2 - x_0^2$ and $(lx) = x_1 + \cdots + x_\rho - 3x_0$. In his prize memoir of 1884, S. Kantor incorrectly asserted the following proposition: any L with integral coefficients, leaving (xx), (lx) invariant, and having the coefficients of x_0 positive and all others negative, must be the description of some planar Cremona transformation. The author approaches the problem by obtaining further properties of an L associated with a C and proves the theorem: *An L is the description of some C if and only if the coefficients are integers, (xx), (lx) are invariants, and the sign of x_0 in all P-characteristics is unchanged.* Some results are listed. (Received January 26, 1944.)

94. Edward Kasner and John DeCicco: *Scale curves in cartography.*

In a non-conformal map T of a surface upon a plane, the scale function $\sigma = ds/dS$ depends upon the lineal-element (x, y, y'). The scale curves defined by $\sigma(x, y, y') = \text{const.}$ are ∞^2 in number. If the osculating circles are constructed to the ∞^1 scales passing through a fixed point P, at P, then the locus of the centers of curvature is a general cubic curve which has a node at P, the directions of the tangent lines at P coinciding with the characteristic directions. A complete analytic characterization of the ∞^2 scales is obtained. A new class of surfaces is found whose ∞^2 scales are all straight. In this class, the only surfaces of constant curvature are the developables. If the ∞^2 scales are of the Lie-Liouville cubic type, then they must form a velocity system. The cubical locus degenerates into a straight line. The velocity systems of scales define a new class of surfaces which are studied in detail. (Received January 25, 1944.)

95. Edward Kasner and John DeCicco: *Scale curves in conformal maps.*

In any conformal map Γ of a surface upon a plane, the scale function $\sigma = ds/dS$ depends upon (x, y) only. Thus there are ∞^1 scale curves $\sigma(x, y) = \text{const.}$ which may be any simple family. If it be demanded that an isothermal (or parallel) system be scale curves of a conformal map Γ such that the gaussian curvature is constant along the scales, then the surface must be developable (Δ) or applicable upon a surface of revolution (Σ). For (Δ), the scales can be any isothermal family, whereas for (Σ), the scales are concentric circles or parallel lines. If the scales are straight or circular such that the gaussian curvature is constant along them, then the surface is a (Δ) or a (Σ), and the scales must be pencils of lines or circles. The application of these results to any conformal map of a sphere upon a plane lead to new characterizations of the Mercator, stereographic, and the general Lambert conical projections. Thus the only conformal map with straight scale curves is the Mercator; and the only circular cases are the stereographic and Lambert maps. (Received January 25, 1944.)

Statistics and Probability

96. Benjamin Epstein and C. W. Churchman: *On the statistics of sensitivity data.*

"Sensitivity data" is a general term for that type of experimental data for which the measurement at any point in the scale destroys the sample. The paper is a generalization of a method of treating such data due to Spearman. (C. Spearman, *The method of "right and wrong cases" (constant stimuli) without Gauss' formulae*, British Journal of Psychology vol. 2 (1908) pp. 227–242.) Formulae for the moments and their standard sampling errors are given. Certain minimization problems are also discussed. (Received January 26, 1944.)

97. E. J. Gumbel: *The observed return period.*

The theoretical return period $T(x)$ of a value equal to, or greater than, x is defined as the inverse of the probability $1-F(x)$. The question is how to calculate, for n observations, the return period $T(x_m)$ of the mth observation $x_m (m=1, 2, \cdots, n)$, and especially $T(x_n)$ of the largest observation x_n for an unlimited variate. This problem is important in probability papers where the variate is plotted as a function of the return period. Engineers use a compromise between the exceedance interval $'T(x_m)=n/(n-m)$ and the recurrence interval $''T(x_m)=n/(n-m+1)$, namely $T(x_m)=n/(n-m+1/2)$. In this case $T(x_n)=2n$. If, however, the probability $F(\ddot{x}_n)$ of the median $\ddot{x}_n$ of the largest value is attributed to x_n, $T(x_n)=1.44n+1/2$. Both methods can hardly be justified. The author attributes the probability $F(\tilde{x}_n)$ of the most probable largest value $\tilde{x}_n$ to x_n. Then $T(x_n)$, as is to be expected, converges toward n, and equals n for the exponential distribution, and $n+1$ for the logistic distribution. In the same way, the probability $F(\tilde{x}_1)$ of the most probable smallest value $\tilde{x}_1$ is used, for an unlimited variate, as frequency of the smallest observation x_1. The frequencies $F(\hat{x}_m)$ of the intermediate $n-2$ observations are obtained by linear interpolation between $F(\tilde{x}_1)$ and $F(\tilde{x}_n)$. Thus the return periods may be determined for all observations. (Received January 27, 1944.)

98. H. E. Robbins: *On the measure of a random set.*

Let X, a measurable subset of Euclidean n-dimensional space E, be a random variable (for example, X may be the set-theoretical sum of N possibly overlapping and independently chosen unit intervals on a line with a given probability distribution for their centers). Let $m(X)$ denote the measure of X, and for any point x of E let $p(x)$ denote the probability that X contain x. Then under very general hypotheses on X it is shown that the expected value of $m(X)$ is equal to the integral over E of $p(x)$. More generally, the expected value of the rth power of $m(X)$ is equal to the integral over rn-dimensional space of the function $p(x_1, \cdots, x_r)=$ probability that X contain all the points $x_1, \cdots, x_r$. (Received January 28, 1944.)