example from the theory of Fourier series that there exists a divergent series $\sum_{n=0} a_n$ such that $\sigma_n^{(2)} - s = 0(n^{-2\alpha'})$, $0 < \alpha' < \alpha < 1$ and $a_n = O(n^{\alpha-1})$. (Received January 5, 1944.)

77. D. V. Widder: *The iterates of the Laplace kernel.*

The iterates of the Laplace kernel $G_0(x, y) = e^{-xy}$ are defined by the recursion relation $G_n(x, y) = \int_0^\infty G_0(x, t) G_{n-1}(t, y)dt$, $n = 1, 2, \cdots$. In an earlier paper (Bull. Amer. Soc. vol. 43 p. 813) the author determined explicitly all these functions which have odd subscripts. They are rational functions of x, y and $\log(x/y)$. In the present paper the iterates with even subscripts are studied. They cannot be expressed in terms of the elementary functions. Complete asymptotic series for their behavior near $x = +\infty$ are obtained. For their behavior near the origin they are developed in power series which converge for all positive values of x. The method consists in expressing the nth iterate in terms of a function $h_n(x)$ whose bilateral Laplace transform turns out to be $\Gamma(-s)^{n+1}\,\Gamma(s+1)^n$. The function $h_n(x)$ is in turn expressible in terms of a new set of transcendental functions related to the familiar exponential integral $EI(x)$. (Received January 14, 1944.)

Applied Mathematics

78. Stefan Bergman: *On solutions of certain partial differential equations in three variables.*

(I) Let $\mathrm{E}(x, y, z, t, \tau)$ be a solution of the equation $\mathrm{G(E)} \equiv \mathrm{L(E)} + [\tau^{-1}U^{-1}(1-\tau^2)(\mathrm{E}_x + i\mathrm{E}_y \cos t + i\mathrm{E}_z \sin t)_\tau] + A[(1/2)\tau^{-1}(1-\tau^2)\mathrm{E}]_\tau$ which satisfies certain boundary conditions. Here $\mathrm{L(E)} \equiv \Delta\mathrm{E} + A(x\mathrm{E}_x + y\mathrm{E}_y + z\mathrm{E}_z) + C\mathrm{E}$, $U \equiv x + iy \cos t + iz \sin t$. Then $\psi(x, y, z) = \int_0^{2\pi}\int_{-1}^{1} \mathrm{E}f[(1/2)u(1-\tau^2), t]/dtd\tau$, where $f(\zeta, t)$ is an arbitrary analytic function of ζ and t, will be a solution of $\mathrm{L}(\psi) = 0$. (II) If A and C are entire functions of $r^2 = x^2 + y^2 + z^2$ alone, then $\mathrm{G(E)}$ becomes $\mathrm{G_0(E)} \equiv (1-\tau^2)\mathrm{E}_{\tau\tau} - \tau^{-1}(1+\tau^2)\mathrm{E}_\tau + r\tau[\mathrm{E}_{rr} + 2r^{-1}\mathrm{E}_r + B\mathrm{E}] = 0$ where $B = [-(3/2)A - (1/2)rA_r + C - (1/4)r^2A^2]$. The author shows that in the case II, there always exists a solution $\mathrm{E} = \mathrm{H}(r, \tau)$ of $\mathrm{G_0(E)} = 0$, which is an entire function of r. The author considers vectors $\boldsymbol{S} = (\psi_1, \psi_2, \psi_3)$ where $\psi_1 = \int_0^{2\pi}\int_{-1}^{1} \mathrm{H}f dtd\tau$, $\psi_2 = i\int_0^{2\pi}\int_{-1}^{1} \mathrm{H}f \cos t dtd\tau$, $\psi_3 = i\int_0^{2\pi}\int_{-1}^{1} \mathrm{H}f \sin t dtd\tau$. Clearly $\mathrm{L}(\psi_K) = 0$, $K = 1, 2, 3$. Let c^1 be a simple closed curve which lies on a sphere $x^2 + y^2 + z^2 = \text{const.}$ If the ψ_K are regular in this sphere then $\int_{c^1} \boldsymbol{S} \cdot d\boldsymbol{X} = 0$. Here $\boldsymbol{X} = (x, y, z)$, and $\cdot$ means the interior product. "Residue" theorems are derived if the ψ_K have singularities in the above sphere. Applications in the theory of waves propagation are indicated. (Received January 28, 1944.)

79. Nathaniel Coburn: *A boundary value problem in plane plasticity for the Coulomb yield condition.*

The following problem is studied: given a semi-plane $x > 0$, composed of plastic material which follows the Coulomb yield condition; the stresses σ_x, σ_y, σ_{xy} acting on the boundary $x = 0$ are considered as known; to find the stresses at any point in the interior of the semi-plane. The method of attack is a modification of that used in studying a similar problem for a perfectly plastic material. The stresses and the functions $\sin 2\gamma$, $\cos 2\gamma$ (where γ is the angle between the x-axis and a tangent to a line of principal shearing stress) are expanded in power series of the friction coefficient. Substituting these power series into the Levy equations, there results an infinite set of Levy equations for the various approximations to the stresses. By requiring that each set of approximations satisfy the equilibrium relations, an infinite sequence of Airy stress functions is obtained. Since each Airy stress function must satisfy two independent Levy equations, there results an infinite set of second order linear partial differential equations. The various approximations are determined and the convergence of σ_{xy} is demonstrated for the case $\sigma_{xy}(0, y)=0$. (Received December 20, 1943.)

80. H. L. Garabedian: *The analogue of Bromwich's theorem for integral transformations.*

The main theorem of this paper provides sufficient conditions in order that a method of summation defined by an integral transformation of the type $z(x) =\int_0^\infty k(x, t)a(t)dt$ shall include Cesàro summability of positive integral order n, defined by the transformation $y(x)=\int_0^x(1-t/x)^n a(t)dt$. A kernel $k(x, t)$ is exhibited which fulfills the conditions of this theorem and thus provides a means of solving in a rigorous fashion the problem of the flow of heat in an infinite rod whose surface is a non-conductor. (Received December 20, 1943.)

81. Michael Goldberg: *A three-space analog of a plane Kempe linkage.*

The following theorem is due to A. B. Kempe: Each of the sides of a plane quadrilateral linkage may be joined by a link to a common pivot without restricting the movability of the linkage. The new pivot associated with a side need not be collinear with the other pivots of that side; the three pivots may form a triangle in a rigid plate. Extensions and further studies on this theorem were made by Kempe (*On conjugate four-bar linkages*, Proc. London Math. Soc. vol. 9 (1878) pp. 133–147) and Darboux (*Recherches sur un système articulé*, Bull. Sci. Math. vol. 3 (1879) pp. 151–192). This paper is concerned with the analogous theorem in three-space. It is shown that it is possible for each of the four bodies of a movable hinged skew quadrilateral (a Bennett linkage) to be joined by a hinged link to a common hinge without restricting the movability. As in the case of the plane, the three hinges in a body need not have a common perpendicular. In general, these hinge lines are non-parallel and non-intersecting. (Received January 29, 1944.)

82. Max Herzberger: *The diapoint characteristic.*

To every point in an optical system with symmetry axis belongs a manifold of *diapoints*, the intersections of the image rays with the meridian plane. If the object point has a sharp image, all the diapoints coincide with it; if the object point has an image with a curved or straight symmetry axis, the diapoints form the axis. The author introduces the diapoint characteristic, a function giving the optical path between point and diapoint. Such a function has a two-dimensional manifold of values in the general case, a one-dimensional manifold in case of symmetry, and is constant if the object point is sharply imaged. The diapoint function $F(x, y, z, v, w)$ can be constructed as a function of the coordinates of the object point (x, y, z) and two parameters v, w. Its analysis for a given system gives at once all the points in space which are sharply or symmetrically imaged, and tells whether there are any surfaces sharply or symmetrically imaged. An example, refraction at a plane and at a sphere, is analyzed. (Received January 24, 1944.)

83. R. P. Isaacs: *Airfoil theory for flows of variable velocity.*

There arise many cases in aircraft practise—conspicuously that of the rotating wing—where the flow over the lift surface is of variable velocity. The first section of this paper deals with theory of an airfoil in a variable speed stream (but constant angle of attack). The result is an integrodifferential relation between the two functions involved: velocity and circulation. Next the case is treated: velocity = constant + sinusoidal variation. Finally the lift is studied and given explicitly for this latter case in the form of a Fourier series. (Received January 24, 1944.)

84. R. v. Mises: *Note on the numerical solution of linear partial differential equations.*

The usual method of numerical solution consists in replacing the differential equation $L(u)=0$ by a set of algebraic linear equations for the values $u(P_i)$ where P_i are the points of a rectangular lattice. This procedure has obvious inconveniences in the case of curved boundaries. Many questions arise if instead of this lattice more general types of point sets are used. The present paper deals with some of these questions, in particular with the number and the arrangement of points required for any order of approximation and with the possibility of applying the method of successive approximations (relaxation method) to the solution of the algebraic equations. Specified formulae are proposed for the case of the Laplace equation. (Received January 28, 1944.)

85. David Moskovitz: *Numerical solution of Laplace's and Poisson's equations.*

This paper considers the system of equations (1) $L_c u_j(i) - u_{j+1}(i) - u_{j-1}(i) = \phi_j(i)$ $(i=1, 2, \cdots, n-1; j=1, 2, \cdots, m-1)$, where L_c is the linear operator defined by $L_c f(i) = cf(i) - f(i+1) - f(i-1)$; c is a constant, the $\phi_j(i)$ are prescribed, and we seek the values $u_j(i)$. The solution of (1) is obtained in symbolic form which is interpreted to define actual solutions. Application is made to the numerical solution of Laplace's and Poisson's equations with prescribed values on a rectangular boundary. Tables of values are included to facilitate making the applications. (Received January 28, 1944.)

86. Isaac Opatowski: *Use of special functions in a problem of uniform strength.*

A cantilever is bent under the action of its own weight and a concentrated load F at the free end. If x is the distance of the cantilever's cross section from its free end, $A(x)$ the area of the cross section, $S(x)$ the section modulus, g the specific weight of the beam, s the maximum stress of each cross section, the condition of uniform strength is: (*) $Fx + g\int_0^x (x-q)A(q)dq = sS(x)$. If the surface bounding the cantilever is representable in the parametric form: $y = u(x)U(t)$, $z = v(x)V(t)$ where (x, y, z) are orthogonal Cartesian coordinates, with z-axis lying in the plane of the bending couple, the use of the functions $w(x) = u(x)[v(x)]^2$, $r(x) = 1/v(x)$ (that is, w directly proportional to the section modulus, r inversely proportional to the radius of gyration) reduces (*) to the differential equation (**) $w''(x) = ar(x)w(x)$, where a is a constant depending on the type of cross section. (**) gives a wide possibility of use of most of the special functions. Cases leading to hyperbolic and cylindrical functions are discussed in detail. The treatment is general, that is, independent of the type of cross section. (Received January 28, 1944.)

87. H. E. Salzer: *Table of coefficients for inverse interpolation with advancing differences.*

This table can be used in place of the similar one described in a previous abstract (49-9-224). In addition it has the advantage that it can be employed for inverse interpolation near the beginning or end of a table and also when only a few tabulated values are available (as is the case, for instance, when solving transcendental equations). The Mathematical Tables Project has computed the coefficients of the products of ratios of advancing differences of various order. These coefficients occur in the formula obtained by the inversion of the Gregory-Newton formula for direct interpolation, employing Lagrange's theorem. The polynomial expressions for those coefficients are given in H. T. Davis, *Tables of the higher mathematical functions*, vol. 1, pp. 80–81. (A slight addition to the formula was made to complete it as far as the eighth order.) The coefficients of the two fourth order and the two fifth order terms were calculated to ten decimals, at intervals of 0.001 of the argument $m=(u-u_0)/(u_1-u_0)$. The coefficients of the four sixth order terms were calculated at intervals of 0.01 and the four seventh order coefficients as well as the seven eighth order coefficients were computed at intervals of 0.1 (all to ten decimals). (Received December 2, 1943.)

88. Andrew Vazsonyi: *On two-dimensional rotational gasflows.*

The differential equation of an inviscous compressible fluid is determined under the condition that the conductivity of the gas is negligible. (By admitting discontinuities this includes flows with shock waves.) The equation of motion is (1) $\psi_{xx}(1-(u^2/a^2))-(2uv/a^2)\psi_{xy}+\psi_{yy}(1-(v^2/a^2))=\rho^2[\partial h_0/\partial\psi-((k-1)/kR)(h_0+q^2/2)\partial s/\partial\psi]$, $u=(1/\rho)\psi_y$, $v=-(1/\rho)\psi_x$, $\rho=e^{-s/R}(h_0-q^2/2)^{1/k-1}$, $q^2(h_0-q^2/2)^{2/k-1}=e^{2s/R}(\psi_x^2+\psi_y^2)$, $a^2=ke^{-(k-1)^{s/R}}\cdot(h_0-q^2/2)$ where the notations are as follows: ψ streamfunction, q velocity, u and v velocity components, ρ density, a local speed of sound, h_0 stagnation enthalpy (Bernoulli constant), s specific entropy, R gas constant, k isentropic exponent. There are two arbitrary functions in this equation, namely: $h_0(\psi)$ and $s(\psi)$; these must be given by the boundary conditions (or by the nature of the discontinuities). The flow is rotational in general and the rotation is given by $\omega=\partial v/\partial x-\partial u/\partial y=-\rho\partial h_0/\partial\psi+(p/R)\partial s/\partial\psi$. For irrotational flow the right-hand side of (1) equals 0. (Received January 28, 1944.)

GEOMETRY

89. Stefan Bergman: *A Hermitian metric and its property.* Preliminary report.

Let the real analytic function $\Phi(z_1, z_2, \bar{z}_1, \bar{z}_2)$, $z_k=x_k+iy_k$, $\bar{z}_k=x_k-iy_k$, $k=1, 2$, of four real variables x_1, x_2, y_1, y_2, satisfy the equation $\Phi=c\{[\partial^2\Psi/\partial z_1\partial\bar{z}_1][\partial^2\Psi/\partial z_2\partial\bar{z}_2]-|\partial^2\Psi/\partial z_1\partial\bar{z}_2|^2\}$, $c=$constant, $\Psi\equiv\log\Phi$, in a domain B of the (four-dimensional) space and become infinite on the boundary of B. The expression $ds_B^2(z_1, z_2)=\sum_{m,n=1}^{2}[\partial^2\Psi/\partial z_m\partial\bar{z}_n]dz_m d\bar{z}_n$ defines in B a Hermitian metric which is invariant with respect to transformations by pairs of analytic functions, $z_k^*=z_k^*(z_1, z_2)$, $k=1, 2$, of two complex variables which are regular in B. Using the methods of the theory of orthogonal functions (see Bergman, *Sur les fonctions orthogonales de plusiers variables complex avec les applications à la théorie des fonctions analytiques*, Interscience Pub-