

## ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

### ALGEBRA AND THEORY OF NUMBERS

53. A. W. Jones: *The lattice isomorphisms of certain finite groups.*

Let  $G$  be any finite group whose lattice of subgroups is modular. The author gives an explicit procedure for obtaining defining relations for all groups whose lattices are isomorphic to the lattice of  $G$ . Since abelian groups have lattices which are modular, the procedure given includes a characterization, for the finite case, of the groups which are lattice isomorphic to abelian groups (cf. Baer, Amer. J. Math. vol. 61 (1939) pp. 1-44). (Received January 28, 1944.)

54. Oystein Ore: *Galois connexions.*

In numerous mathematical theories there occur order inverting correspondences between two structures  $P$  and  $Q$ ,  $p \rightarrow Q(p)$ ,  $q \rightarrow P(q)$ , such that  $PQ(p) \supseteq p$ ,  $QP(q) \supseteq q$ . The basic properties of such Galois correspondences are derived. They correspond to a duality between structures or closure relations and can also be considered to be mappings of closure relations. They are closely connected with the theory of binary relations as discussed in the author's Colloquium Lectures given in Chicago in 1941. Among the applications should be mentioned a general Galois theory for binary relations, illustrated in detail for the case of equivalence relations. (Received December 7, 1943.)

55. Raphael Salem: *On a remarkable class of algebraic integers. Proof of a conjecture of Vijayaraghavan.*

Let us denote by  $C$  the set of all algebraic integers such that all their conjugates have moduli inferior to 1 ("Pisot-Vijayaraghavan numbers" or briefly "P. V. numbers"). It is proved that the set  $C$  is closed.  $C$  being enumerable it follows that it is (1) nowhere dense; (2) not dense in itself; (3) reducible. There exists a number larger than 1 which is the smallest "P. V. number." (Received December 29, 1943.)

56. P. M. Whitman: *Identities in lattices of equivalence relations. Preliminary report.*

It is shown that free lattices can be represented by equivalence relations. Therefore no lattice identity can hold in every lattice which consists of all equivalence relations on some set, unless it holds in every lattice. (Received December 27, 1943.)

57. P. M. Whitman: *Lattices and equivalence relations*. Preliminary report.

It is shown that any lattice is isomorphic to a sublattice of the lattice of all equivalence relations on some set. (Received January 28, 1944.)

#### ANALYSIS

58. Jesse Douglas: *Separable transformations with separable inverse*.

All transformations  $X=f(x)+h(y)$ ,  $Y=g(x)+k(y)$  are found whose inverses are of the same form. Six essentially different types are obtained. If  $x, y$  are interpreted as minimal coordinates  $u+iv, u-iv$  (and  $X, Y$  similarly), we have all harmonic transformations whose inverses are harmonic. The paper will be published in full. (Received January 15, 1944.)

59. K. O. Friedrichs: *The identity of weak and strong extensions of differential operators*.

In applying the theory of linear operators in Hilbert spaces or spaces  $L_p$  to the solution of differential equation problems, it is impossible to retain the meaning of differentiation in the ordinary sense; the concept of differential operator must be extended. Two such extensions offer themselves, a "weak" and a "strong" one, that is, the adjoint of the "formal-adjoint" and essentially the closure. The purpose of the paper is to prove the identity of these two extensions for general linear differential operators. The main tool for the proof is a certain class of smoothening operators approximating unity. They yield the identity of both extensions immediately for differential operators with constant coefficients; they are a strong enough tool to yield this identity likewise for operators with non-constant coefficients. (Received December 3, 1943.)

60. B. M. Ingersoll: *On singularities of solutions of linear partial differential equations*.

Let  $U(z, \bar{z})$ ,  $z=x+iy, \bar{z}=x-iy$ ,  $x, y$  real, be a real solution of  $L(U) \equiv \Delta U + AU_x + BU_y + CU = 0$ , where  $A, B$ , and  $C$  are entire functions when  $x$  and  $y$  are extended to complex values. To every such solution corresponds uniquely a complex solution  $u(z, \bar{z}) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} z^m \bar{z}^n$  of  $L(U) = 0$ , with the property that  $\sum_{n=0}^{\infty} A_{0n} \bar{z}^n = \pi U(0, 0) \exp(-\int_0^z a(0, \bar{z}) d\bar{z})$ , where  $a(z, \bar{z}) \equiv (1/4) \{ A [ ((z+\bar{z})/2, (z-\bar{z})/2i) ] + iB [ ((z+\bar{z})/2, (z-\bar{z})/2i) ] \}$ . These solutions were introduced by Bergman (Rec. Math. (Mat. Sbornik) N. S. vol. 2 pp. 1169-1198 and Trans. Amer. Math. Soc. vol. 53 pp. 130-155) who showed that the location of the singularities of  $u(z, \bar{z})$  is determined by the sequence  $\{A_{m0}\}$ . Employing this result the author investigates the relations between sequences  $\{A_{mk}\}$ ,  $k$  fixed,  $m=0, 1, 2, \dots$ , and the positions of singularities of  $u(z, \bar{z})$ . For example, using a result of Mandelbrojt (C. R. Acad. Sci. Paris, 1937, pp. 1456-1458) he determines the arguments of the singularities on the circle of convergence of  $u(z, \bar{z})$  in terms of the sequence  $\{A_{mk}\}$ ,  $k$  fixed. In the last section of the paper, using explicitly an integral representation of the complex solutions  $u(z, \bar{z})$ , the author investigates the real solutions  $U(z, \bar{z}) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} D_{mn} z^m \bar{z}^n$  of  $L(U) = 0$ . He constructs, in terms of  $\{D_{mk}\}$ ,  $k$  fixed,  $m=0, 1, 2, \dots$ , and some of the deriva-