

LOGIC AND FOUNDATIONS

333. A. R. Schweitzer: *On a class of ordered $(n+1)$ -ads relevant to the algebra of logic. III.*

The author analyzes the formal sum (now not necessarily reflexive) of $(n+1)$ -ads of generalized constituent type into two types of ordered dyads: $\xi \cdot \eta$, multiplicative, and $(\xi) + (\eta)$, additive, where $(\xi) = \xi_1 \xi_2 \cdots \xi_{n+1}$. Addition is assumed commutative and associative; multiplication is associative but not necessarily commutative. Formal addition is extended to elements ξ by assuming the formal equivalence: $(\alpha_1 + \lambda_1)(\alpha_2 + \lambda_2) \cdots (\alpha_{n+1} + \lambda_{n+1}) = \alpha_1 \alpha_2 \cdots \alpha_{n+1} + \lambda_1 \alpha_2 \cdots \alpha_{n+1} + \cdots + \lambda_1 \lambda_2 \cdots \lambda_{n+1}$. This assumption effects a gradual transition from the author's theory to formal developments based on the distributive property such as Grassmann's extensive algebra, the algebra of logic, Frobenius' calculus of group elements, number systems (fields) and so on. Grassmann's algebra contrasts with the latter in reference to closure properties. These are defined in terms of completeness (extension, invariance, attainment of limit of development) of the fundamental set of elements with reference to adjunction of operations on elements ξ , for example, $\xi \cdot \eta$, $\xi + \eta$. Finally the author discusses applications of restricted distributive properties such as $(x+y)y = xy + y^2$, $(x+I)y = xy + Iy$, where $Iy = y$ and addition and multiplication may or may not be reflexive. Reference is made to a paper reported in this Bulletin, abstract 48-3-135. (Received August 15, 1942.)

334. Saly R. R. Struik: *Axiomatics of affine geometry.*

The axiomatics are developed for two and three dimensions separately. The theorem of Desargues is used as an axiom in two dimensions. (Received August 6, 1942.)

STATISTICS AND PROBABILITY

335. T. N. E. Greville: *Regularity of label-sequences under configuration transformations.*

There is developed a class of transformations on sequences of arbitrary labels in terms of which a wide variety of problems in the theory of probability can be formulated. It is shown that, with mild restrictions on the transformations used and on the measure function assumed on the label-space, almost every label-sequence produces a transform having the frequency distribution expected. The class of transformations considered is shown to include as special cases the four fundamental operations of von Mises: place selection, partition, mixing, and combination. (Received August 4, 1942.)

336. H. B. Mann: *The construction of orthogonal Latin squares.*

A Latin square is based on a group G if in the reduced form its rows form a regular representation of G . A set of orthogonal squares is based on G if all the squares of the set are based on the same representation of G . The mappings S_1, S_2, \cdots, S_r of G onto itself are r -fold complete if every element of G is of the form $X^{S_i + S_{i+1} + \cdots + S_{i+h}}$ for every i and h with $1 \leq i \leq r-h$ where X^{S_i} is the image of X under the mapping S_i and $X^{S_i + S_j} = X^{S_i} X^{S_j}$. A set of r orthogonal squares based on G exists if and only if G admits an r -fold complete mapping and vice versa. No $4n+2$ sided Graeco-Latin square based on a group exists. The orthogonal set is constructed by the automorphism method if for every i the mapping $S_1 + S_2 + \cdots + S_i$ is an automorphism of G . If

c_q is the number of classes of elements of order q in G , then not more than $\min c_q$ orthogonal squares can be constructed from G by the automorphism method. (Received September 30, 1942.)

337. Abraham Wald: *On a statistical problem arising in the classification of an individual in one of two groups.*

Let π_1 and π_2 be two p -variate normal populations which have a common covariance matrix. A sample of size N_i is drawn from the population π_i ($i=1, 2$). Denote by $x_{i\alpha}$ the α th observation on the i th variate in π_1 , and by $y_{i\beta}$ the β th observation on the i th variate in π_2 . Let z_i ($i=1, \dots, p$) be a single observation on the i th variate drawn from a population π where it is known that π is equal either to π_1 or to π_2 . The parameters of the populations π_1 and π_2 are assumed to be unknown. It is shown that for testing the hypothesis $\pi=\pi_1$ a proper critical region is given by $U \geq d$ where $U = \sum \sum s^{ij} z_i (\bar{y}_j - \bar{x}_j)$, $\|s^{ij}\| = \|s_{ij}\|^{-1}$, $s_{ij} = [\sum_{\alpha} (x_{i\alpha} - \bar{x}_i)(x_{j\alpha} - \bar{x}_j) + \sum_{\beta} (y_{i\beta} - \bar{y}_i)(y_{j\beta} - \bar{y}_j)] / (N_1 + N_2 - 2)$, $\bar{x}_i = (\sum_{\alpha} x_{i\alpha}) / N_1$, $\bar{y}_i = (\sum_{\beta} y_{i\beta}) / N_2$ and d is a constant. The large sample distribution of U is derived and it is shown that U is a simple function of three angles in the sample space whose exact joint sampling distribution is derived. (Received August 7, 1942.)

338. Jacob Wolfowitz: *On the consistency of a class of non-parametric statistics.*

Let X and Y be two stochastic variables about whose distribution nothing is known except that they are continuous and let it be required to test whether their distribution functions are the same. Let V be the observed sequence of zeros and ones constructed as described elsewhere (Wald and Wolfowitz, *Annals of Mathematical Statistics*, vol. 11 (1940), p. 148). Suppose that the statistic $S(V)$ used to test the hypothesis is of the form $S(V) = \sum \phi(l_j)$, where l_j is the length of the j th run and $\phi(x)$ a suitable function defined for all positive integral x . The notion of consistency, originated by Fisher for parametric problems, has already been extended to the non-parametric case (loc. cit., p. 153). The author now proves that, subject to reasonable conditions on $\phi(x)$ and statistically unimportant restrictions on the alternatives to the null hypothesis, statistics of the type $S(V)$ are consistent. In particular, a statistic discussed by the author (*Annals of Mathematical Statistics*, vol. 12 (1942)) and for which $\phi(x) = \log(x^x/x!)$ belongs to the class covered by the theorem. (Received August 7, 1942.)

TOPOLOGY

339. O. G. Harrold: *A higher dimensional analogue of a theorem of plane topology.*

Since the carriers of a Vietoris cycle may have a dimensionality far removed from that of the cycle, it is of interest to determine a class of spaces for which the bounding cycles have membranes of dimensionality exceeding that of the cycle by unity. An example is known of an $1c^1$ carrying an essential 1-cycle which has a 1-dimensional carrier but bounds only on a 3-dimensional set. A similar example is constructed in the Euclidean space E_6 . That such cannot happen in certain Euclidean spaces is indicated by the following theorem, which is a generalization of a known result for $n=0$. Let X be a compact $1c^n$ subset of E_{n+2} . Denote by F the frontier of X relative to E_{n+2} . There exists in X a compact subset X_0 which is $1c^n$ such that $X_0 \supset F$ and $\dim X_0 \leq n+1$. (Received August 4, 1942.)