surfaces, proving the general Morse relations. The integrals to be made stationary have the form $\iint\{f(X, Y, Z)+k(X^2+Y^2+Z^2)^{1/2}\}\,dudv$ where X, Y, Z are the Jacobians $(y_u, z_u; y_v, z_v)$, and so forth, k is a positive constant, and f satisfies the following four conditions: (1) f is positively homogeneous in X, Y, Z; (2) $\epsilon=f(X, Y, Z)-\{Xf_{X_0}+Yf_{Y_0}+Zf_{Z_0}\}\geqq0$; (3) $f(X, Y, Z)\geqq m>0$ for $X^2+Y^2+Z^2=1$; and (4) $f(X, Y, Z)<k$ for $X^2+Y^2+Z^2=1$. The first three are usual conditions to be expected; the main restriction is (4) which asserts that the integral is dominantly an area integral. The problem is to find extremal surfaces bounded by a given rectifiable curve. The integral is replaced by $\iiint f(X, Y, Z)dudv+kD[x]$ where $D[x]$ is the Dirichlet integral, and the extremal surfaces found are given in isometric representation. (Received August 5, 1942.)

323. J. A. Shohat: *An inequality for definite integrals*. Preliminary report.

Starting with a simple case of the Schwarz inequality, another inequality is derived in a very elementary manner. This inequality seems to be new and may be considered an improvement over the Schwarz inequality. In fact, it is shown that the new inequality yields better results than the Schwarz inequality (it reduces to the latter, in some special cases).—Applications are indicated to orthogonal functions and to polynomials. (Received September 17, 1942.)

APPLIED MATHEMATICS

324. Lipman Bers and Abe Gelbart: *On solutions of the differential equations of gas dynamics*. I.

By a transformation due to Chaplygin and Busemann the equations of the steady two-dimensional irrotational flow of a gas can be reduced, in the subsonic case, to the form (1): $u_x=\tau(y)v_y$, $u_y=-\tau(y)v_x$, where τ is a given function. Equations of the same form occur in other branches of mechanics of continua. The class of complex-valued functions $f(x+iy)=u+iv$, where u and v satisfy (1) is shown to have many properties in common with analytic functions. Two processes, one the inverse of the other, are introduced: τ-differentiation and τ-integration. The τ-derivative and the τ-integral of f belong to the class. By τ-integrating a complex constant, a_n, n times a function $a_n \cdot Z \cdot {}^n(z)$, of the class is obtained. From these "powers," "polynomials" and "power series" are formed. A "polynomial" of the nth degree possesses n zeros. Any function, f, of the class can be developed in a unique manner in a power series, the "coefficients" a_n, being given by the nth τ-derivatives of f. Domains of convergence, mapping properties, and particular functions are considered. Analogous methods are applied to more general types of equations; in particular to the system $u_x=\tau_1(y)v_y$, $u_y=-\tau_2(y)v_x$ which describes the subsonic as well as supersonic flow. (Received August 27, 1942.)

325. Garrett Birkhoff: *A reversibility paradox in hydrodynamics*.

It is shown that every known differential equation and general boundary condition describing compressible non-viscous flow is reversible, in the sense that it is compatible with reversal of the direction of flow without change in pressure. This is even true in the presence of a conservative force-field. It follows that, contrary to a widespread impression, any theory of non-viscous flow (airfoil theory, resistance to projectiles, or resistance to surface craft in water) which purports to be based entirely on

general considerations, must predict that a flow and its reverse give identical pressure distributions, hence identical lift and drag. This reversibility paradox partly overlaps the d'Alembert paradox, but it is far simpler, and more general in that it applies to modern airfoil theory and to ships. Special methods for avoiding it under particular circumstances, and their comparative validity, are then discussed. (Received August 27, 1942.)

326. Nathaniel Coburn: *The linear yield condition in the plane plasticity problem*.

The curvilinear nets consisting of curves of maximum and minimum shearing stress have been studied, extensively. However, all of these studies have assumed that the material under investigation follows the von Mises yield condition. In this paper, the above nets are investigated for materials which follow a linear yield condition. It is shown that two cases exist: (1) corresponding to lines of slope ± 1; (2) corresponding to lines of slope other than 0, ± 1 (the case of slope 0 is that due to von Mises). For the first case, it is shown that the two families of curves of the net possess the following two properties: (1) they constitute an equiareal system; (2) at any given point, the magnitudes of their radii of curvature are equal. Analytically, the functional relation is determined which the metric coefficients satisfy. For the second case, it is shown that a generalization of the von Mises diagonal property is valid. Analytically, the determination of the metric coefficients is shown to depend upon the solution of a nonlinear second order partial differential equation. Special solutions are studied. (Received September 28, 1942.)

327. W. R. Foote, Hillel Poritsky, and J. J. Slade: *The motion of a rotor with unequal shaft flexibilities mounted in bearings of unequal flexibility*.

In this paper is discussed the motion of a rotor possessing different flexibilities in two mutually perpendicular directions and mounted in bearings which likewise possess different flexibilities. The equations of motion of such a rotor revolving at constant speed form a fourth order linear system with periodic coefficients. The passage from stable to unstable solution is investigated in detail. For a fourth order system, unlike the second order Mathew equation, this passage may (and does) occur not only through periodic solutions when the characteristic exponent vanishes (or is equal to i) but also by having some of the characteristic exponents equal to each other. What are believed to be new and more convenient methods of calculation are developed. Some of these readily carry over to any order system with periodic coefficients which are trigonometric polynomials. (Received August 4, 1942.)

328. R. P. Isaacs: *Application of polygenic functions to two-dimensional elasticity*.

Let σ and τ be the tensile and shearing stresses in a two-dimensional body on a lineal element at z and inclined at θ with the real axis. If all internal parts of the body are in equilibrium, $\sigma + i\tau$ is the derivative of a polygenic function g which satisfies $I[G_z] = 0$, and the derivative of any such function is a stress distribution in equilibrium. The derivative circle of g is the Mohr circle. $G_{\bar{z}zz} = 0$ is necessary and sufficient for the body to obey Hooke's law. (Received September 12, 1942.)