

studies have considered 100, 500, or once in England (to refute Lombroso's theory) 1500 cases. But from the correct statistical standpoint, far more cases are needed to establish a law. Over a period of years, an attempt has been made to use statistical methods in the study of penological problems in the Massachusetts Reformatory for Women, but the results will take on real significance and be conclusive only when similar investigations are made all over the United States. (Received August 1, 1942.)

289. D. S. Villars: *Significance tests for multivariate distributions.*

The observed mean of sets of m variates, each normally and independently distributed, is distributed around the population mean according to a χ^2 distribution with m degrees of freedom. The sum of squares of deviations of n observed points from the observed mean is distributed as χ^2 with $m(n-1)$ degrees of freedom (not with $n-1$). A much more powerful test for correlation than that by the correlation coefficient is described, which for bivariate distributions, involves comparisons between $n-1$ and $n-1$ degrees of freedom. This can be extended to $m-1$ tests with m variates. Distribution of distance between two means and distribution of fiducial radius is worked out in detail for two variates. (Received July 30, 1942.)

TOPOLOGY

290. D. W. Hall: *On a partial solution of a problem of J. R. Kline.*

As a partial solution of a problem of J. R. Kline, the following theorem is established. In order that a compact locally connected continuum M be homeomorphic with a sphere it is necessary and sufficient that it satisfy the following conditions: (a) no two points separate M , (b) for every simple closed curve J in M the set $M-J$ has at least two and at most a finite number of components. (Received June 22, 1942.)

291. W. M. Kincaid: *On non-cut sets of locally connected continua.*

This paper is concerned with certain generalizations of the well known result that corresponding to any non-cut point p of a space S which is a locally connected continuum, an arbitrarily small region having a connected complement and containing p can be found. It is shown that any closed non-cut set P of such a space S can be imbedded in the sum R of a finite number of regions (lying in a preassigned ϵ -neighborhood of P) so chosen that $S-R$ is a locally connected continuum. If, in addition, there exists a family of sets \mathcal{F} no element of which separates $S-P$, then another set R' , contained in R and having the same properties, can be found such that no element of \mathcal{F} contained in $S-R$ separates $S-R'$. If the elements of \mathcal{F} are single points, the sets R and R' can be replaced by a single set having the properties of both. Further results are obtained in the special case where S is not separated by any m points. (Received July 24, 1942.)

292. R. G. Lubben: *Mappings of spaces H Fréchet on completely regular spaces.*

Let T be a space H Fréchet, K be the aggregate of all completely regular Hausdorff decompositions (Alexandroff and Hopf, *Topologie*, p. 70; the space of this decomposition is to be a completely regular Hausdorff space) of T into mutually exclusive point sets, and \sum be the sum of the elements of K . If $T \supset M$ and for $G = \bar{G} \subset T - M$ there exists a function which is continuous over T , takes on values from zero to unity, and

is identical to zero over M and to unity over G , then it is said that M is completely T -regular. (1) A set is T -regular if and only if it belongs to Σ . (2) K is the totality of upper semi-continuous decompositions of T into completely T -regular sets. (3) For A and B elements of K let " $A < B$ " mean that each element of A is a subset of an element of B ; this ordering of the elements of K defines a complete lattice, L , which has properties analogous to those of the " $\delta(S)$ " studied by the author (Transactions of this Society, vol. 49 (1941), pp. 459–463). If S is a regular Hausdorff space and T is the space of its atomic decomposition points, then L and $\delta(S)$ are isomorphic. (Received August 1, 1942.)

293. Deane Montgomery and Hans Samelson: *Groups transitive on spheres.*

Let G be a compact connected Lie group which is transitive and effective on the n -sphere S^n . If n is even it is shown that G must be simple. If n is odd then G must be essentially the product of two simple groups one of which is either the identity, the circle group, or the group of quaternions of absolute value unity. There is given a fairly complete analysis of exactly what groups can be transitive on S^n . Speaking in general terms the results show that in the main only groups well known to be transitive on S^n can be transitive on S^n . There are one or two results on the structure of rotation groups. (Received July 9, 1942.)

294. Deane Montgomery and Hans Samelson: *Groups transitive on the n -dimensional torus.*

It is shown that if G is a compact connected Lie group which acts effectively and transitively on the n -dimensional torus, then G is the n -dimensional toral group. (Received July 9, 1942.)

295. Deane Montgomery and Leo Zippin: *A class of transformation groups in E_n .*

If a compact connected topological group acts on euclidean n -space and if it has at least one $(n-1)$ -dimensional orbit, then it is known that all orbits except one are $(n-1)$ -dimensional. It is shown here that this exceptional orbit must be a point and that there is a closed set which touches each orbit precisely once and is homeomorphic to a ray. The $(n-1)$ -dimensional orbits are all homeomorphic and in many homology and homotopy properties they resemble $(n-1)$ -spheres. (Received June 12, 1942.)

296. Hassler Whitney: *Complexes of manifolds.*

In geometric applications of combinatorial topology, it is very convenient to be able to realize chains and cochains with the help of geometric figures. These figures may not appear naturally subdivided into complexes; but they will in general be made up of manifolds, joined together like the cells of a complex. These "complexes of manifolds" are studied thoroughly here. The most general ones (which have fairly simple intersection properties) are more general than complexes, even in the neighborhood of a point. The "locally flat" ones are also studied. The most helpful tools are vectors and cones. (Received July 28, 1942.)

297. Hassler Whitney: *The self-intersections of an M^n in E^{2n} .*

Let the differentiable n -manifold M^n be mapped regularly by f into E^{2n} ; by a slight

deformation, suppose that the self-intersections are isolated. The space \mathfrak{X} of ordered pairs of points of M may be considered as bounded by the space \mathfrak{S} of directions in M at all points of M . Let n be even, and M , orientable. To f corresponds F , mapping \mathfrak{X} into E^{2n} ; the signed self-intersections of f give the signed zeros of F , and their algebraic number I_f is half d_F , the degree of F on \mathfrak{X} over the origin, or on \mathfrak{S} about the origin. If f_0 is deformed into f_1 , each f_i being regular, then $I_{f_1} = I_{f_0}$. There exist mappings with any desired I_f . If n is odd or M is non-orientable, all holds if one reduces mod 2 and identifies "opposite points of \mathfrak{X} and \mathfrak{S} ." M^n may be imbedded (without self-intersections) in E^{2n} . (Received July 28, 1942.)

298. Hassler Whitney: *The singularities of an M^n in E^{2n-1} .*

Let M be a piece of an orientable n -manifold M^n , bounded by A . Let f be a mapping of M into E^{2n-1} , such that the singular points (points where f is not regular) are isolated and a certain condition (see this Bulletin, abstract 48-9-269) holds at each. Let f be one-one in A . Let $n \geq 3$ be odd. Then if $\mathfrak{L}(M)$ is the looping coefficient $\text{LC}(A, A') = \text{Kronecker index KI}(A, M)$ (A' being A pushed slightly into M), this measures the signed number of singularities of f in M . Hence if $\mathfrak{L}(M) \neq 0$, f cannot be regular. For M closed, of course $\mathfrak{L}(M) = 0$. All holds for n even if reduced mod 2. For n even, there is a regular mapping f of an n -cube in $E^n \subset E^{2n-1}$ into E^{2n-1} , arbitrarily near (but not with derivatives) to the identity, equal to the identity on A , and with $\mathfrak{L}(M) = 2$. Arbitrarily near any mapping f of M^n into E^{2n-1} there is a regular mapping; thus M may be immersed in E^{2n-1} . (Received July 28, 1942.)