### 266. Hermann Weyl: *On Hodge's theory of harmonic integrals.*

Hodge's fundamental existence theorem for harmonic integrals on Riemannian manifolds of any dimensionality is proved by the parametrix method. (The proof incorporated in Hodge's recent book on *Harmonic Integrals*, Cambridge, 1941, is wrong.) (Received July 1, 1942.)

### 267. Hassler Whitney: *Differentiability of the remainder term in Taylor's formula.*

If $f(x)$ is of class $C^m$, and $1 \leqq n \leqq m$, then $f(x) = \sum_{i=0}^{n-1} f^{(i)}(0)x^i/i! + x^n f_n(x)/n!$. It is shown that $f_n(x)$ is of class $C^{m-n}$, but not necessarily of higher class, and $\lim_{x\to 0} x^k f^{(m-n+k)}(x) = 0$ $(k=1, \cdots, n)$. A converse is true. A similar theorem holds in more dimensions. (Received July 28, 1942.)

### 268. Hassler Whitney: *Note on differentiable even functions.*

It is shown that an even function $f(x)$ of class $C^{2s}$ (or class $C^\infty$, or analytic) may be written as $g(x^2)$, with $g$ of class $C^s$ (or class $C^\infty$, or analytic). (Received July 28, 1942.)

### 269. Hassler Whitney: *The general type of singularity of a set of $2n-1$ smooth functions of n variables.*

Let $f$ be a mapping of class $C^1$ of an $n$-manifold $M^n$ into an $M^{2n-1}$. Then arbitrarily near $f$ is a mapping $f'$, regular except at isolated singular points; at each of these, a certain condition (C) holds. (C) involves first and second derivatives, but is independent of the coordinate system employed. If (C) holds at $p$, and the mapping is of class $C^{4r+8}$ (or class $C^\infty$, or analytic), then coordinate systems about $p$ and $f(p)$, of class $C^r$ (or class $C^\infty$, or analytic), exist such that the mapping is exactly $y_1 = x_1^2$, $y_i = x_i$, $y_{n+i-1} = x_1 x_i$ $(i=2, \cdots, n)$. (Received July 28, 1942.)

## Applied Mathematics

### 270. Stefan Bergman: *Operators in the theory of differential equations and their application.* I.

By introducing $u = x\cos\theta + y\sin\theta$, $v = -x\sin\theta + y\cos\theta$ and $\xi = (\sigma/2k) + \theta$, $\eta = (\sigma/2k) - \theta$, where $\sigma_x = \sigma + k\sin 2\theta$, $\sigma_y = \sigma - k\sin 2\theta$, $\tau_{xy} = -k\cos 2\theta$ the equations of the theory of plasticity can be written in the form $(\partial^2 u/\partial\xi\partial\eta) - u/4 = 0$, $(\partial^2 v/\partial\xi\partial\eta) - v/4 = 0$ (see Geiringer and Prager, Ergebnisse der exakten Naturwissenschaften, vol. 13, p. 350). Here $\sigma_x, \sigma_y, \tau_{xy}$ are stresses, $x, y$, cartesian coordinates. Particular solutions of these equations can be written in the form $u(\xi, \eta) = \int_{-1}^{1} \exp\,(t(\xi\eta)^{1/2})\{f[\xi(1-t^2)/2] + g[n(1-t^2)/2]\}(1-t^2)^{1/2}dt$ where $f$ and $g$ are arbitrary twice continuously differentiable functions of one variable. (See Duke Mathematical Journal, vol. 6 (1940), pp. 538 and 557.) This class of functions possesses a base $\{u_\nu(\xi, \eta)\}$ such that each $u_\nu$ satisfies two (simple) ordinary linear differential equations of second order with rational coefficients. Entire solutions $u_\nu$ of the above partial differential equation are such that every $u$ defined in a convex domain can be approximated by sums of the form $\sum_{\nu=1}^{n} a_\nu^{(n)} u_\nu$. The author indicates an approximation procedure of a function $u$ given by its boundary values. These functions $u$ possess singularities which can be characterized in a way analogous to that in Comptes Rendus de l'Académie des Sciences, vol. 205 (1937), pp. 1360–1362. (Received June 3, 1942.)

271. Stefan Bergman: *Operators in the theory of partial differential equations and their application.* II.

Let $v(x, y)e^{i\theta(x,y)}$ denote the velocity vector of an irrotational steady flow of compressible fluid. Let $\zeta=\Lambda(v)+i\theta$, $\bar{\zeta}=\Lambda(v)-i\theta$, where $d\Lambda(v)/dv=[1-M^2]^{1/2}/v$, and $M=v/[d_0^2-(1/2)(k-1)v^2]^{1/2}$, $d_0$ and $k$ being constants. Finally: let $E^* =1+t\zeta^{1/2}Q(\zeta, \bar{\zeta}, t\zeta^{1/2})$ where $Q(\zeta, \bar{\zeta}, p)$ is an (arbitrary) solution of $Q_{p\bar{\zeta}}+2p(Q_{\zeta\bar{\zeta}}+FQ) +2F=0$, and $Q$ is supposed to be an odd function of $p$. Then $\psi(v, \theta) =Re\{\int_{-1}^{1}T(\zeta+\bar{\zeta})E^*(\zeta, \bar{\zeta}, t)f[(1/2)\zeta(1-t^2)]dt/(1-t^2)^{1/2}\}$ where $f$ is an arbitrary analytic function of one complex variable is the stream function of a suitable subsonic flow, and the stream function of every flow can be represented in the above form. $T$ and $F$ are suitable functions of $(\zeta+\bar{\zeta})$. Using this result the author proves that various sets of particular solutions $\{p_\nu (v, \theta)\}$ are complete. The author indicates a method of determining the constants $a_\nu^{(n)}$ in $\psi_n=\sum_{\nu=1}^{n}a_\nu^{(n)}p_\nu$ in such a way that $\psi_n$ approximates the flow in a channel which is given in the $xy$-plane (physical plane), provided that the image of the flow in the hodograph plane is schlicht. The method is a generalization of one given in Duke Mathematical Journal, vol. 6 (1940), p. 537. A similar procedure can be applied in the case of a supersonic flow. (Received July 28, 1942.)

272. Vladimir Morkovin: *On the deflection of anisotropic thin plates.*

Deflections $w$ of an anisotropic plate (with one plane of elastic symmetry) bounded by an analytic curve $C_0$ are considered. The general solution of the differential equation for $w$ is known to be expressible in terms of two analytic functions $f_1(z_1)$ and $f_2(z_2)$, where the complex variables $z_1$ and $z_2$ are related to the variable $z_0$ of the original plane by $z_k=p_kz_0+\bar{q}_k\bar{z}_0$, the constants $p_k$ and $q_k$ depending on the material of the plate. (See S. N. Lechnitzky, Journal of Applied Mathematics and Mechanics, (n. s.), vol. 2 (1939), pp. 181–210.) Transformations $z_k=\omega_k(\zeta_k)$ are found which make any point on $C_0$ correspond to points $e^{i\theta}$ on the circumferences $\gamma_k$ of unit radii in new $\zeta_1$ and $\zeta_2$ planes having the same polar angle $\theta$, and which are conformal in some neighborhoods of $\gamma_k$. Then the functions $\phi_k(\zeta_k)\equiv f_k(z_k)$ can be determined from the two given boundary conditions if these are expressed in terms of $e^{i\theta}$. A detailed solution illustrating this general procedure is carried out in the case of a clamped elliptic plate with polynomial loading. (Received July 31, 1942.)

## Geometry

273. H. S. M. Coxeter: *A geometrical background for the description of de Sitter's world.*

This paper begins with an elementary treatment of the process by which an elliptic or hyperbolic metric in the plane at infinity of affine space induces a Euclidean or Minkowskian metric in the whole space. The various kinds of sphere are defined, and are seen to provide models for non-Euclidean planes, including the "exterior-hyperbolic" plane which is a two-dimensional de Sitter's world. (See Eddington, *The Mathematical Theory of Relativity*, 1924, p. 165.) Then comes a simple proof of Study's theorem to the effect that one side of a triangle is greater than the sum of the other two, and finally a discussion of some cosmological paradoxes. (Received July 31, 1942.)

274. J. J. DeCicco: *New proofs of the theorems of Beltrami and Kasner on linear families.*

Here new proofs of the theorems of Beltrami and Kasner on linear families of