pupil P. Pepper who worked out proofs for theorems stated by Minkowski.

As will be seen from this description, the book restricts itself entirely to Minkowski's own work in the geometry of numbers. Professor Hancock explains in the introduction that this was done in order to limit the content of the book which otherwise would have been beyond bounds. Still, one may regret that the newer developments were not at least indicated. There is another factor which decreases the value of the book, this being that the representation often is not as good as one may wish (compare, for instance, Article 8 with the corresponding section of the *Diophantische Approximationen*). The different parts of the book could have been connected more closely. Finally, there are numerous misprints some of which are confusing. In the opinion of the reviewer, it would not be surprising, if many readers should prefer the original texts. On the other hand, there will be many mathematicians who will be very grateful to Professor Hancock for facilitating for them access to Minkowski's beautiful investigations.

RICHARD BRAUER

Enzyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band I, Teil 1, Heft 2, 114 pp.; Band I_1, Teil 1, Heft 4, 51 pp.; Band I_1, Heft 5. 54+54+28 pp. Leipzig and Berlin, Teubner, 1939.

This new edition of the Enzyklopädie der Mathematischen Wissenschaften appears exactly forty years after the publication of the first volume of the first edition in 1899. The original project of compiling and presenting a comprehensive review of the science of mathematics and its allied fields was considered a monumental and ambitious task which aroused great interest among contemporary mathematicians. The initiative to the Enzyklopädie was taken by Felix Klein, Heinrich Weber and Franz Meyer and a great number of other prominent mathematicians was gradually associated with this initial group. To begin with, the work had been planned in the form of a regular encyclopedia in which the material should appear as special articles for each mathematical term. After an early attempt along these lines it became clear that this method of presentation led to considerable overlapping and the artificial classification of the subjects according to the alphabet tended to make them incoherent and lacking in general views. This prompted the fortunate decision of giving a systematic account of the field of mathematics in which the various articles on the subdivisions of the science were fitted into their natural connections as far as it was possible. It may also be remarked that although the original encyclopedia was to a large extent created and dominated by German mathematicians, the plan was supposed to be at least partially international in character and certain volumes of a French edition, slightly different from the German version, were actually published.

It took nearly forty years to complete the first edition of the encyclopedia and many of its parts, particularly those which were printed in the earliest volumes, have long outlived the usefulness for which they were intended and they now appear more as scientific historic documents than reviews of their subjects. The new edition proposes to remedy this situation. Already from the few numbers which are available it is clear that the plan of the new edition is almost identical with the former one. The old articles have been thoroughly rewritten and brought up to date, and new articles have been put in whenever recent developments have warranted it. But the new edition, in spite of its many strong points also seems destined soon to develop the deficiencies of the first.

The intention of these remarks is of course not to criticize the general idea of a mathematical encyclopedia. Clearly a modern and authoritative review of the state of the various branches of mathematics and some of its most closely allied fields is highly desirable both as a tool for the working scientist and as a reference for information in general. Such a comprehensive presentation cannot be replaced by the numerous accounts of special fields which have recently become so popular, perhaps in some measure just because the encyclopedia has been so deficient.

With the previous experiences in mind it is evident that the project of an encyclopedia, if it is to fill the needs for which it is intended, must be on a permanently self-renewing basis. The best form of such an undertaking is of course wide open to debate and discussion. However it would appear absolutely necessary to base the work on an encyclopedia perhaps not so very different from the previous one, but with the strong proviso that the several volumes on the various branches of mathematics should be published simultaneously as far as possible so that the various parts would be approximately in the same state of actuality. Presumably one would have to have an editorial body of specialists for each field. This group should be responsible both for the initial volumes and also for the decision when they were to be revised. After a period of ten to fifteen years most accounts would probably have to be rewritten. But equally important is the problem of keeping the reports up to date in the meanwhile as far as possible, and this is the main point in which the suggested scheme would differ from the present encyclopedia. Supplemental reports would have to be issued at fairly regular intervals, for instance every second or third year. These bulletins need not give any new reviews, it would be sufficient if they contained the references to those new papers on the subject which had appeared in the intervening period.

It is not necessary to point out the advantages of such an encyclopedic institution could it be initiated. It is quite evident to anyone engaged in mathematical research and familiar with the bibliographical investigations which are necessary in order to establish the actual state of some mathematical domain. Anyone engaged in advanced instruction will have observed the notable difficulties many of the beginning, younger men have in gathering their material. Perhaps it is not too optimistic to believe that some efforts could be saved, some disappointments avoided or even some publication costs reduced by minimizing the duplication which does not occur too rarely in mathematical literature.

In spite of the present all-time lowest ebb in international scientific collaboration it is quite evident that for the publication of such a permanent mathematical encyclopedia international cooperation would be almost imperative and in any case extremely desirable. I do not believe it would be too great a task for the American Mathematical Society, which has already taken the lead in so many mathematical enterprises, to sponsor such a publication. The Society has among its members more qualified and outstanding mathematicians than any other similar body in the world so that it would have no dearth of competent collaborators even if it were to carry the burden alone. The cooperation of the Mathematical Reviews would also greatly facilitate the compiling of periodic bulletins.

The economic basis for such a project is another question which would have to be carefully studied. The first encyclopedia was extremely expensive and it received its main support through the subscription of the libraries. Relatively few individuals could afford to own the whole encyclopedia. Even separate volumes were not commonly sold and the purchase of such single volumes does not seem to have been particularly encouraged. For a new publication the support of the libraries could of course be counted upon to continue. But it appears probable that one could also successfully urge the subscription to separate parts and their supplements by individuals. The number of research workers in mathematics at the present time is many times larger than it was in 1900, and there is every reason to believe that this number will continue at the same high level or even that it will increase. If the encyclopedia could be established as an ever ready research tool it is not unreasonable to expect that a considerable group of mathematicians could be induced to take partial subscriptions in their own special fields. Such an arrangement would materially contribute to the economic foundation of the encyclopedia.

Let me conclude by saying these are of course thoughts for the future and for times of total peace. But when the present urgent tasks have been attended to, it may not prove to be a utopian scheme.

After this long preamble let us turn to the new edition of the encyclopedia which occasioned it. The three numbers at hand afford a typical example of the differences one is confronted with in the various topics. The first part on the number system and general set theory deals with a field which in many ways has acquired a settled form. The second part on group theory covers a well-developed domain still going strong and with an overwhelming number of detailed contributions. Somewhat similar is the situation in the ideal theory considered in the third part. But here in the final article on "Verbände" or lattices one encounters a new field which is in such a state of rapid growth that even a review written a couple of years ago cannot be considered to give a satisfactory account of its present status.

The article on the foundations of the number system by F. Bachmann gives in clear and quite broad form the axiomatic foundations for the concept of a number. The axioms given by Peano as well as Dedekind's introduction of the integers by means of chains and set-theoretical considerations serve as the foundations for the theory of natural numbers. From this starting point one finds a survey of the various paths which have been followed in order to obtain the negative, the rational, the real and the complex numbers. The Dedekind cut method and Cantor's fundamental sequences and papers on these subjects are discussed at some length. The formal introductions of the complex numbers and some of the characterizations of the real and the complex number fields form the concluding parts of the article. It may be observed in regard to the complex numbers that the historical side of their development is curiously lacking, the only names of early mathematicians dealing with them, which have been mentioned, are those of Gauss, Hamilton and Cauchy. Perhaps it is the intention to fill this lacuna in connection with the article on the theory of equations.

The related chapter on representation of real numbers by limiting processes is written by K. Knopp. Here one finds a discussion of the classical method of exhaustion and its more modern counterpart, the Cauchy sequences. The representations of real numbers by infinite series, infinite products and continued fractions are rather summarily treated since these topics will be considered in special articles later. Among the representations of real numbers which are not so commonly used one may mention the Cantor series and the representations by principal fractions studied particularly by Engel and Lüroth. The goodness of approximations is touched upon and in the last few pages one finds a discussion of the criteria for irrationality. The transcendency of real numbers will be dealt with in a separate article.

The general theory of sets, limited to those parts of set theory into which the topological concepts do not enter, is very ably presented by E. Kamke. After the basic operations of set theory, including limit sets and the Suslin scheme, have been introduced, one obtains a general view of the theory of cardinal numbers with their various formal properties. Of particular interest is the author's discussion of the questionable points in set theory. The difficulty in obtaining a suitable definition for sets is illustrated by the antinomies, which the author proposes to eliminate by a reference to the theory of types. The axiom of choice is considered at length with special mention of the numerous investigations by Sierpinski. The author prefers the acceptance of a definition of sets which actually includes the axiom of choice. Certain authors have also questioned the tacit assumption of the existence of the power set of a given set, but there does not seem to be any logically objectionable consequences to this assumption. The various axiomatic foundations for set theory, particularly those given by Zermelo, Brouwer and Russel-Whitehead are considered. The final and longest part of the exposition is devoted to ordinal numbers, well-ordering, transfinite induction and related questions. The tendency of the last few years to eliminate the use of transfinite induction by means of general principles, for instance Zorn's lemma, came too late to be included.

In the article by W. Magnus on general group theory the author has achieved a *tour de force* by condensing into 50 pages a review of practically all that has been done in group theory since this important branch of mathematics was created. A modern survey of group theory of this bibliographical kind did not exist till now and the article will certainly prove to be a valuable aid for anyone who wishes to establish what has been done in connection with any particular problem. It is also the kind of exposition which one should like to see kept up to date in the future.

On the other hand the extremely condensed presentation makes this chapter much less readable than the preceding ones. There does not seem to be any advantage in going into details of this chapter in this review. It falls in four main parts: (1) the general concepts of group theory, (2) structure of groups with finite chain conditions, (3) finite groups, (4) construction of groups, infinite groups. The fourth part contains a brief account of the topological groups, while Lie groups will be presented in some other connection. The references seem to be complete up to the year 1937 with a few later papers mentioned.

The abstract ideal theory and some of its most important applications are discussed by W. Krull in two consecutive articles. The first review is divided into two parts, one giving the general decomposition theory and dimension theory for ideals, the other containing the multiplicative ideal theory which is closer to the ordinary ideal theory for algebraic numbers. In this latter part one finds a discussion of such important concepts as integrally closed rings and evaluation theory. The final divisions of the article deal with the extension of norms, degrees, ramification theory including differents, conductors and discriminants to general rings. This article covers a field in which the author himself has made some of the main contributions.

The second article by Krull on ideal theory covers its applications to polynomial ideals and elimination theory. The nucleus of the polynomial ideal theory is to be found in the theorems of Lasker and Hilbert as well as the results of van der Waerden. The elimination theory contains the theory of resultants and the various elimination methods for homogeneous and nonhomogeneous systems. Of importance is also the general theory of multiplicities due to van der Waerden and its applications to the theory of singularities of algebraic manifolds.

In a last, joint article Hermes and Köthe review the theory of "Verbände" or lattices. One finds first an exposition of the fundamental properties of lattices and then in several divisions some of the most important branches of the theory. One chapter deals with the theory of Dedekind lattices, the theorem of Jordan-Hölder and its extensions and the main decomposition theorems of algebra. Next follows the theory of distributive lattices and Boolean algebras including Stone's representation theory and some of the applications of Boolean algebras to topology and logic. A final chapter contains the theory of complemented lattices with special accounts of the works of Birkhoff, Menger and von Neumann. The article is clear and the presentation covers the material capably and satisfactorily as far as it goes, but, as we have already indicated, this is a typical example of a field in which a review cannot remain up to date for any considerable period. The account seems to have been written in the beginning of the year 1938 and to supplement it a list of papers published in 1938 and 1939 has been appended. But even so, a great number of contributions have already been made since this time, some of them of such importance that they could not be omitted even in a general survey.

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