$+y_1)]/(y_0-y_1)$. If $\partial(\int Q dy)/\partial x$ exists, then each integrating factor of $Pdx+Qdy=0$ has a similar differentiability property. (Received May 20, 1942.)

244. J. D. Rommel: *On conservative transformations of functions of two variables.*

Necessary and sufficient conditions are given in order that $y(s, t)=\int_0^\infty\int_0^\infty k(s, t, \sigma, \tau)x(\sigma, \tau)d\sigma d\tau$ shall transform $x(\sigma, \tau)$ such that $\lim_{\sigma,\tau\to\infty}x(\sigma, \tau)$ exists into $y(s, t)$ such that $\lim_{s,t}y(s, t)$ exists. From this result a characterization of regularity is obtained. By considering step-kernels and step-functions, regular transformations of double sequences are characterized. (Received April 18, 1942.)

245. Raphael Salem and D. C. Spencer: *The influence of gaps on density of integers*

An infinite sequence of integers is said to have the complete gap property with respect to $\omega(x)$ ($\omega(x)$ being a positive non-decreasing function for $x\geqq 0$) if in every closed interval $(a,a+l)$ $(a\geqq 0,\ l\geqq l_0)$ there exists an open interval of length not less than $\omega(l)$ which contains no point of the sequence. Such a sequence will be denoted by $S[\omega(x)]$. Let $\nu(m)$ be the number of terms of the sequence not greater than m. The following theorems are proved: (1) If the integral $\int_0^\infty(\omega(x)/x^2)dx$ diverges, $\lim \nu(m)/m=0$. (2) If $\omega(x)$ is such that $\omega(x)/x^2$ decreases and the integral converges, there exists an $S[\omega(x)]$ with $\liminf \nu(m)/m>0$. (3) If $\omega(x)=\theta x$ $(0>\theta>1/3)$, then $\nu(m)/m=O(m^{-\alpha})$ where $\alpha=[\log (1-2\theta)/(1-3\theta)]/[\log 2(1-2\theta)/(1-3\theta)]$. (4) If $0<\theta<1/3$, there exists a sequence $S[\theta x]$ with $m^{-\alpha}=O(\nu(m)/m)$. (5) If $\omega(x)=\theta x$ $(1/3\leqq\theta<1/2)$, then $\nu(m)=O(\log m)$. (6) If $1/3\leqq\theta<1/2$, there exists a sequence $S[\theta x]$ with $\log m=O(\nu(m))$. (7) If $\omega(x)\geqq x/2$, there exists no infinite sequence $S[\omega(x)]$. (Received April 30, 1942.)

Applied Mathematics

246. H. K. Brown: *The resolution of boundary value problems by means of the finite Fourier transformation.*

The finite sine transformation and the finite cosine transformation are defined as the linear functional operations $\mathfrak{S}\{G\}=\int_0^\pi G(x)\sin nxdx=g_s(n)$ and $\mathfrak{C}\{G\}=\int_0^\pi G(x)\cdot\cos nxdx=g_c(n)$, respectively. The inversion of the product of the transforms of G and H can be made by means of four Faltung theorems. The finite sine transformation was applied to a problem in general heat flow in one dimension in which the nonhomogeneous linear partial differential equation has coefficients which may be functions of the time. It was proved in detail that this problem can be resolved into a standard heat flow problem which has a differential equation of simpler type and constant coefficients. The Faltung theorems permitted an inversion in closed form of the solution of the transformed boundary value problem. A general heat flow problem of similar type in three dimensions was resolved into the same standard problem in one dimension. By the introduction of a fundamental set of solutions of the transformed problem it was possible to make a resolution of the boundary value problems of a general vibrating string, membrane and beam. (Received April 11, 1942.)

Geometry

247. C. J. Everett: *Affine geometry of vector spaces over rings.*

Methods of E. Artin's *Coordinates in affine geometry* (Publication of the University of Notre Dame, Reports of a Mathematical Colloquium, (2), Issue 2) are used to show