

BOOK REVIEWS

Research—A National Resource. II. Industrial Research. (Report of the National Research Council to the National Resources Planning Board.) Washington, D.C., United States Government Printing Office, 1941. 11+370 pp. \$1.00.

This report, based on a canvass of industrial laboratories, gives an account of the nature and extent of present day industrial research in this country. It consists of an organized collection of over twenty studies, prepared by specially qualified men, covering such topics as the development of industrial research, fundamental research in industry, careers in research, research in the aeronautics, petroleum and steel industries, location and extent of research, and research abroad. There are studies on the role of chemistry, of physics, biology, mathematics, and various fields of engineering, in industrial research. The part on mathematics (pp. 268–288), prepared by Dr. T. C. Fry, will be found interesting and valuable to all mathematicians.

Dr. Fry's report, entitled *Industrial mathematics*, has been reprinted as a supplementary issue of the American Mathematical Monthly (vol. 48 (1941), no. 6, part 2), and also in the Bell System Technical Journal (vol. 20 (1941), pp. 255–292). It begins with a discussion of the nature of the industrial mathematician's work, followed by a list of the qualifications that these men should possess, pointing out that the mathematician in industry must generally function as a consultant rather than as a project man. The lack of a center of training for such men is stressed. In recent months steps have been taken to improve this situation. A study of the uses of mathematics in industry and a section on statistics make up the rest of the report. The many interesting examples give a vivid impression of the growing use of mathematics in industry.

R. V. CHURCHILL

The Analysis of Economic Time Series, by Harold T. Davis. (Cowles Commission for Research in Economics, Monograph no. 6.) Bloomington, Indiana, Principia Press, 1941. 14+620 pp. \$5.00.

It is unfortunate and a sign of the extreme specialization of science and the scientists that many advances made in related fields are ignored or neglected by the mathematicians. This is not altogether their fault, because nobody could possibly follow the ever growing literature in all phases of science utilizing mathematics. The whole mathematical profession and especially its statistical branch is in-

debted to Professor H. T. Davis, who has in this monumental volume assembled and lucidly interpreted a great number of the recent developments in the interesting, if somewhat puzzling, field of the study and analysis of economic time series. It also contains many of his own very important contributions. I feel certain that this book is destined to exert a deep influence upon the future development of the field. It also will stimulate the thinking of many who have been working on entirely different problems and who may find some of their difficulties related to the questions discussed in Professor Davis' book.

The first chapter deals with the history of the analysis of time series and gives a very valuable account of the most important writings in the field. It also affords an excellent introduction to the other parts of the book. The mathematical reader may not feel he can even glance through the whole 600 pages of the book. But he will be able to gain a rapid survey of the field and to select the material most valuable to himself by reading carefully the 60 pages of the first chapter.

The second chapter deals with the technique of harmonic analysis; this is the representation of empirical data by Fourier series. It does not on the whole add very much to the classical treatment of the problems. The last section relates the technique of harmonic analysis and more generally the method of decomposing empirical series by orthogonal function to the theory of multiple correlation. It contains some very valuable and original ideas. There are also, as in all other chapters, applications to economic data, which will be of great interest to economists.

The third chapter treats serial correlation. This is the phenomenon of the appearance of leads and lags among correlated variables. A special case is auto-correlation, where the individual items of one time series are correlated with each other. This chapter presents the best exposition of these extremely interesting and difficult problems which have given so much headache to the statistician. A great deal of the work originally done by Norbert Wiener is reproduced but important theorems are added. There are also new calculation techniques. The problem of continuous spectra is treated as well as some questions connected with random variation which have received so much attention lately. Finally, this chapter gives an application of the ideas of lag correlation to supply and demand curves, which leads to a discussion of the so-called cobweb theorem well known in economic statistics.

The fourth chapter brings an extensive discussion of the important problem of the theory of random series, which naturally is of great

interest not only to professional statisticians but also to workers outside the field of economic statistics. It has a definite relationship to some of the fundamental assumptions of probability theory, for instance, to the frequency definition of probability by von Mises. Goutereau's method is discussed and Yule's theory of random series is generalized with the help of the auto-correlation function. The theory of accumulated random series which has received some attention from mathematicians recently is presented as well as the influence of moving averages upon random series. The theory of sequences and reversals originally introduced by Bortkiewicz and recently further developed by H. E. Jones receives considerable attention. Finally, an application to the economic problems of stock market action is given.

Chapter 5 deals with the difficult and puzzling problem of degrees of freedom in economic time series, which has troubled statisticians for a long time. It is well known to the economic statistician that additional observations within the same time interval do not in general add materially to our information about economic phenomena we are studying with the help of statistical methods. If, however, the Fisherian theory of degrees of freedom is uncritically applied, it would seem that we are gaining a great deal of information. Professor Davis attacks the question as a problem in inverse probability essentially on the basis of Bayes' theorem. He considers particularly Schuster's, Walker's and Fisher's tests of significance in harmonic analysis and some problems recently discussed by Thurston, Frisch, and Hotelling in the field of factor analysis. Dissatisfied with previous results he finally proposes a very ingenious method which he calls the method of elementary energies.

Chapter 6 deals with the analysis of trends. The secular trend represents the non-periodic movements in economic time series. The discussion adds some new formulae for the correlation of residuals from polynomial trends. It treats the elimination of seasonal variation by the method of link relatives and the variate difference method which utilizes series of successive finite differences of the data. A very interesting section deals with the fitting of the logistic, which is a somewhat difficult problem because of the nature of the curve. Its solution is, however, important for social and economic statistics because of the role of the logistic in population theory and related fields. Hotelling's method of fitting the curve to its differential equation is discussed extensively as well as other approaches. Additional sections deal with the growth of population and of production. Finally Mr.

Leavens' interesting theory of frequency distributions of time series is presented.

Chapter 7 deals with periodogram analysis. This is one particular technique for the search for hidden periodicities. It utilizes the amplitudes of sine-cosine curves fitted to trial products. Davis presents some material on sine-cosine series, random series, smoothed and cumulated random series, and economic applications to various empirical data. A section deals with sunspot numbers and another with galvanometer series. It is a clear and valuable presentation of the technique of periodogram analysis.

Chapter 8 deals with the evidence and explanation of economic cycles. This is the much discussed and still largely unsolved problem of the somewhat irregular appearance of booms and depressions. Professor Davis first discusses and classifies economic business cycle theories and treats in more detail the mathematical theories of Evans, Roos, Frisch, Tinbergen, and Kalecki. He also goes into some problems on statistical hysteresis which are very interesting from a purely mathematical point of view. He discusses the methods proposed by Lotka, Volterra, Jones, and others in this field. The remainder of the chapter is devoted to some original work in the theory of crises which Professor Davis tries to explain by a sort of resonance theory. Chapter 9 deals with the nature of wealth and income and presents first statistical data on the personal distribution of income as well as the classical theory of Pareto's law. Professor Davis also tries to give a new and very interesting general distribution function which seems to fit the empirical income distribution better than Pareto's function or variants proposed by other writers. The chapter deals, however, mainly with economic and social problems.

Chapter 10 is entitled, "The dynamic trends from the point of view of the equation of exchange," and will not be of very great interest to non-economists.

Chapter 11 takes up very courageously the difficult problem of forecasting economic time series, especially as applied to stock prices. It reviews the classical work of Henry Schultz, *The standard error of a forecast from a curve*. Then it presents Professor Davis' own contribution which essentially assumes that it is not possible to forecast in the future beyond a range equivalent to the range of the available data. There are some interesting ideas about moving periodogram analysis and the method of probable error bands.

The last chapter finally deals with interpretation and critique of the material presented, especially from the point of view of business

cycles. It also gives a new "economic" interpretation of history and presents some interesting analogies between mathematics and economics.

This book should prove very useful to the statistician as a handbook. He will find a great deal of stimulation and valuable material and procedures even if he is not professionally concerned with economic problems. The pure mathematician can gain a survey of the field. He may be interested especially in some of the unsolved problems of serial correlation and periodogram analysis which evidently require very powerful tools of analysis. One has to be extremely grateful to the author for having accumulated so much material and presented it in such a readable and interesting form.

GERHARD TINTNER

A Survey of Modern Algebra. By Garrett Birkhoff and Saunders MacLane. New York, Macmillan 1941. 11+450 pp. \$3.75.

In this book Professors Birkhoff and MacLane have made an important contribution to the pedagogy of algebra. Their emphasis is on the methods and spirit of modern algebra rather than on the subject matter for itself. The word "survey" in the title is quite accurate; for, although many topics are treated, none of them is really completely developed. The most important parts of each theory are included and that is all that can be asked of an introductory textbook.

Because of the authors' emphasis on "method" rather than "fact" the book will not be of much use as a reference work. But there is no dearth of good reference works in algebra, and in the reviewer's opinion the present textbook will prove more useful than another encyclopedic treatise would have been.

A discussion of the topics included will help to indicate the authors' purposes.

The first three chapters of the book are ostensibly devoted to the development of number systems: starting with postulates for the integers, then defining rational numbers in terms of the integers, and next giving an outline of the Dedekind construction of real numbers. Actually much more is happening. Such fundamental concepts as congruence, residue class, isomorphism, and ordered and well ordered sets are introduced and applied in a natural manner to the theory. Also the generalizations from integers to integral domain, and from rational number to field are made at suitable stages of the development. This procedure of starting with properties of a familiar mathematical system and generalizing to an abstract system is typical of