78. J. W. Peters: *The euclidean geometry of the n-dimensional simplex.*

In this paper theorems associated with the triangle and the tetrahedron are extended to the n-dimensional simplex formed by $n+1$ points in a euclidean space of n dimensions. The centroid and Monge point of the simplex as well as the centroids and Monge points of the faces are defined. The following extension of Mannheim's theorem for a tetrahedron is proved. The $n+1$ planes determined by the $n+1$ altitudes of the simplex and the Monge points of the corresponding faces meet in the Monge point of the simplex. A hypersphere on the centroids of the faces is discussed. It is shown that this hypersphere passes through $3(n+1)$ points associated with the simplex and has a number of properties similar to the nine point circle associated with a triangle and with the twelve point sphere associated with a tetrahedron. (Received November 8, 1941.)

79. J. L. Vanderslice: *Invariant theory of vector pencil fields.*

At each point $(x^1, \cdots, x^n)$ of a space subject to general coordinate transformations is associated a pencil of contravariant vectors, $\xi^i(x^1, \cdots, x^n, u)$. The parameter u of the pencil is normalized to give an invariant metric parameter x^0 associated with each coordinate system x^i and transforming like the gauge variable of generalized projective geometry. The principal result is the discovery of an affine connection with components which are functions of x^α $(\alpha=0, 1, \cdots, n)$ and a method of covariant differentiation of tensor functions of x^α. A study of the equivalence problem then leads to a complete set of tensor invariants for the vector pencil field. (Received November 17, 1941.)

80. André Weil and C. B. Allendoerfer: *A general proof of the Gauss-Bonnet theorem.*

The following generalized Gauss-Bonnet theorem was recently proved independently by Allendoerfer and Fenchel: If a closed Riemann manifold R_n of even dimension can be made a subspace of an euclidean space then $\int KdO = 1/2\omega_n\chi$ where K is the total curvature of the manifold, ω_n is the area of an n-dimensional sphere, and the integration is over the manifold. The present paper removes the restriction that R_n be a subspace of an euclidean space. To do this R_n is subdivided into simplices each of which is small enough to have an isometric euclidean imbedding. The method of tubes is applied to these subdivisions separately, special attention being paid to their boundaries. The terms resulting from the boundaries are found to be intrinsic and drop out when the simplices are reassembled to form R_n. (Received November 26, 1941.)

Logic and Foundations

81. E. C. Berkeley: *Application of symbolic logic to punch card operations.*

This paper discusses the analysis of operations with punched tabulating cards, and also to some extent operations with handwritten cards, as taking place in a large life insurance company, for purposes of valuing policies, computing, recording, and summarizing payments, and so on. The chief instrument of analysis is a system of coding, constructed using symbolic logic and other techniques. The system of coding is exhibited in part, and examples of the coding are given. Some related problems of practical and theoretical nature are described and discussed. (Received November 21, 1941.)

82. G. D. W. Berry: *On formalizing semantics.*

Two alternative formalizations of semantics may be distinguished. Quine's *Mathematical Logic* (New York, 1940) with modifications, provides the logical and syntactical components of each. In each, designation is primitive, denotation and truth defined. Besides including statements specifying that all designated expressions are elements, that only abstracts designate, and that nothing designates more than one thing, the axioms of both systems include all statements formed from 'If — is designated, then '—' designates it' by replacing the blanks with any constant term. The paradoxes of Grelling, Richard, König, and Epimenides each requires an hypothesis of the form, '— is designated.' In the first system, which has only one designation-relation, each paradox becomes a *reductio ad absurdum* argument for the existence of an undesignated entity. The argument of the Epimenides, in particular, establishes the existence of non-Tarskian statements, or statements equivalent to the denial of their truth. Some axioms of the form '— is designated' are adopted. Others may be added piecemeal, when needed, at the investigator's risk. In the second system, which has infinitely many designation-relations, each statement of this form is an axiom for some appropriate designation-relation. Classes undesignated and statements non-Tarskian relative to one such relation are respectively designated and Tarskian relative to another. (Received November 26, 1941.)

83. Alonzo Church: *On sense and denotation.*

The title is intended to translate Frege's *Über Sinn und Bedeutung.* The *denotation* of a proper name (including descriptions, class abstracts, also sentences as proper names of truth-values) is that of which it is a name. One may *understand* a proper name in the sense of knowing its meaning linguistically, yet not know its denotation. This linguistic meaning is the *sense.* In particular, the sense of a sentence is the proposition. Two proper names coinciding in sense must have the same denotation—although to determine the denotation, given the sense, may require settling a question of extra-linguistic fact. If a constituent part of a proper name is replaced by another having the same sense, the sense of the whole is not altered; if it is replaced by another of the same denotation, the denotation of the whole is not altered, but the sense may be. Thus is solved, in particular, Russell's puzzle about "the author of Waverley." Assuming tacitly that names have only one kind of meaning, the denotation, Russell concluded that a logically sound language cannot employ descriptions: sentences containing descriptions must be reconstrued as mere convenient abbreviations of sentences of a different form. This seems to be tenable but is less elegant that Frege's solution of the same problem. (Received November 21, 1941.)

84. Nelson Goodman: *Sequences.*

The only method hitherto available for defining sequences on the basis of class theory results in identifying a sequence of n components with a certain class n types higher than those components. By this method, the sequence $x_1 \cdots x_n$ becomes: $\hat{z}\{(z \in 1: z \subset x_1 . \vee . x_1 \subset z . x_1 \neq \Lambda) \vee (z \in 2: z \subset x_2 . \vee . x_2 \subset z . x_2 \neq \Lambda) \vee \cdots (z \in n: z \subset x_n . \vee . x_n \subset z . x_n \neq \Lambda)\}$. The method works for a sequence of finite length n in a universe containing at least $n+1$ elements of the type of members of components of the sequence. That the components must be classed can be shown to be no actual restriction. Any component x_k of a sequence Q is defined as: $\hat{z}\{z \in p`(K \cap Q) . \vee . p`(K \cap Q)$

$=\Lambda . z \in s`(K \cap Q)\}$. The above schemata may be supplanted by formal definitions of the class of sequences and of the kth component of any sequence. Every class of classes is correlated to some one sequence by a definable relation we call the "establishment" of the sequence by the class. A class that establishes a sequence having the members of the class, and the null class, as its components is called "self-ordered." (Received November 21, 1941.)

85. S. C. Kleene: *On the interpretation of intuitionistic number theory.*

A closed existential statement $(Ex)A(x)$ can be interpreted from the constructivist standpoint as a partial communication of a more explicit statement which gives a number x such that $A(x)$ holds, together with such further information as is required to complete the meaning of $A(x)$ for that x. Likewise, a closed generality statement $(x)A(x)$ can be interpreted as the assertion of the possibility of describing an effective general method for obtaining, to any given x, such information as will complete the meaning of $A(x)$ for that x. These ideas are made precise in a truth-definition for the class of statements formed from given general recursive predicates of natural numbers by the operations of the predicate calculus. The definition will be used to obtain some metamathematical results for the intuitionistic predicate calculus and number theory. Typical clauses, for closed statements: If $A(n)$ is realized by a, $(Ex)A(x)$ is *realized* by $2^n \cdot 3^a$. $(x)A(x)$ is *realized* by the Gödel number e of a general recursive function $\phi(x)$ such that, for every n, $\phi(n)$ realizes $A(n)$. $A \rightarrow B$ is *realized* by the Gödel number e of a partial recursive function $\phi(x)$ such that, whenever a realizes A, $\phi(a)$ realizes B. Then A is *realizable*, if there is a number a which realizes it. (Received November 21, 1941.)

86. Barkley Rosser: *The Burali-Forti paradox.*

Four basic principles of the theory of ordinal numbers and well-ordered series are: (1) To every well-ordered series there corresponds a unique ordinal number. (2) The series of ordinal numbers is well-ordered. (3) If x is a term of a well-ordered series S, then the series consisting of all terms of S which precede x is also well-ordered, and has a smaller ordinal number than S. (4) Any ordinal number, α, is the ordinal number of the series of all ordinals which precede α. The Burali-Forti paradox is the statement that these four principles are incompatible. To avoid the paradox, two devices have been tried. One is to adopt some form of a theory of types which will invalidate (4). The other device has the effect of invalidating (1) in certain critical cases. In his book, *Mathematical Logic*, Quine proposed a device which is essentially a combination of the two aforementioned devices. Unfortunately Quine's device does not invalidate (4) and fails to invalidate (1) at certain critical points, and so the Burali-Forti paradox appears in Quine's book. This is shown in detail in the present paper. (Received November 24, 1941.)

87. A. R. Schweitzer: *On the genesis of the algebra of logic in the foundations of geometry.*

Following his theory of geometrical relations (American Journal of Mathematics, vol. 31 (1909), p. 400), the author separates descriptive n-space ($n=1, 2, 3, \cdots$) into 2^{n+1} compartments $C(\ )$, $C(\alpha_i)$, $C(\alpha_i\alpha_j)$, $\cdots$, $C(\alpha_1\alpha_2 \cdots \alpha_{n+1})$ by means of two n-simplexes T_1 and T_2 such that $T_1=\alpha_1\alpha_2 \cdots \alpha_{n+1}$ with sides prolonged is in the interior of, and cut off by, T_2. Then $\sum(I)$ denotes the reflexive formal sum of all C's; $\sum(\alpha_1)$ is the sum of all C's containing α_1 as argument, and so on; $\sum(\lambda_1)$ is the sum of all C's not containing α_1, and so on. Then $C(\ )=\sum(\lambda_1\lambda_2\cdots\lambda_{n+1})$, $C(\alpha_1)=\sum(\alpha_1\lambda_2\cdots\lambda_{n+1})$ and so on. The C's are replaced by their corresponding expressions in terms of $\sum$ and the subsequent development follows that of an earlier paper reported in this Bulletin (abstract 47-9-430). The preceding provides a representation R_n for the latter exposition; R_2 is equivalent to the so-called Euler diagram which in turn is topologically equivalent to the figure of a triangle in a descriptive plane extended in accordance with the author's system ${}^2K_2$ (ibid., p. 395, Axiom 12). R_2 is topologically equivalent to the figure of a trifoliate curve inscribed in a circle. Another basis for the algebra of logic in the author's theory is found in a simple modification of his axioms for a linear "permutation" (ibid., p. 370). (Received November 24, 1941.)

Statistics and Probability

88. R. D. Gordon: *An application of the Cauchy integral to a problem in probability theory.*

The author was consulted about interpretations of certain data on biological populations. The problem aims at obtaining error corrections on the data. The classical basis of the Bayes-Laplace formula is chosen: $(x)(n/x)=(n)(x/n)$ where the notation has the usual meaning in probability theory. The probability (n/x) is known through its generating function. The distribution (x) of the parameter x, an integer, is taken on reasonable grounds to be $(x)=1/b+1$, for $0\leqq x\leqq b$; $(x)=0$ otherwise. Then $(n)=\sum(x)(n/x)$ is determined. An "observation" results in a value n_0 of the stochastic variable n. The problem: to determine the distribution $(n/n_0)=\sum(n/x)(x/n_0)$. We actually obtain the generating function for the *moments* of (n/n_0). Simple manipulations result in the Cauchy-integral equation $(x)G_{(n/x)}(t;\ x)=1/2\pi i\int_{(\Gamma)}G_{(n)}(t/s)\cdot E(s;x)ds/s$ where the G's represent generating functions for the probability distributions indicated in their subscripts, and $E(s;\ x)$ represents an *unknown* generating function for probabilities (x/n) *with respect to n*. The problem is solved by applying standard generating-function operators to this equation and integrating over the circle $|s|=t$. (Received November 14, 1941.)

89. G. F. McEwen: *On the probability that a ratio of random numbers will depart from a harmonic ratio by less than a given amount.*

The ratio of two numbers is called harmonic if it equals the ratio of two small integers, one figure numbers, for example. In practical applications both numerator and denominator are subject to error and their ratio accordingly departs from perfect harmony. Accordingly, it is necessary to estimate the probability P of getting by chance a ratio departing less than a given amount from perfect harmony. This problem of determining the proportion, among all possible combinations of numbers, that have a ratio departing from perfect harmony less than this amount, is equivalent to that of finding the probability P that the ratio of any two numbers drawn at random will depart by an amount X or less from the harmonic ratio N/M. The solution is $P=2(M^2/N)X/[1-(M/N)^2X^2]$, where $N>M$. (Received October 27, 1941.)

Topology

90. W. W. Flexner: *Noncommutative chains.* II. Preliminary report.

Extending the author's recent work (Duke Mathematical Journal, vol. 8 (1941), pp. 497–505) to a finite convex 3-complex, 2-dimensional noncommutative chains and