Mathematical Logic. By W. V. Quine. New York, Norton, 1940. 8+348 pp. $4.00.

Unfortunately, the system of logic presented in this book admits the Burali-Forti paradox. This admission renders the system inconsistent. As a result the book fails in its primary purpose and will need serious revision.

Not all portions of the book are affected by the paradox. In particular, the first three and last of the book's seven chapters can survive unchanged (except for very minor details in Chapter 7). Also, much valuable material can be salvaged from the three affected chapters, notably major portions of §§38–40, 43, 44, 47–52.

Quine's first chapter deals with the propositional calculus, the second with the theory of quantification, and the third with certain aspects of the theory of classes, including the theories of identity and description. The treatment of these subjects is very thorough, with good explanations, so that for these subjects the book can be recommended either as a text or as a reference book.

Quine's seventh chapter contains a new proof of Gödel's theorem on the existence of undecidable propositions. The novel feature of Quine's proof is the following. Let L be a logic with a denumerable number of symbols. We can think of L as a system involving only two symbols, namely "x" and an accent, by replacing the denumerable symbols of L by x, x', x'', x''', $\cdots$. So there is no loss of generality in assuming that L has a finite number of symbols, namely $S_1, S_2, \cdots, S_n$. Let expressions of L be finite sequences of the S's (allowing multiple uses). Let us endow $Mxyz$ with the meaning: if x is a single symbol, then x is the next symbol after y in the list $S_1, S_2, \cdots, S_n$; if x is a complex expression, then x is the result of writing y followed by z; if x is not an expression then $x=y$. Then, as Quine shows in detail, a symbolism based on M, four variables, an accent (for producing more variables), the stroke function, and parentheses (for universal quantification) suffices for the usual syntactical discussions about L. In particular, it suffices to define "provable formula" for L. If we take the S's to be the nine symbols of the M-system, then the M-system suffices for its own syntax, and the Gödel theorem follows readily.

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