author, upon carefully re-examining these paradoxes, has reached the following conclusions: (a) None of them arises if the would-be paradox maker is estopped from employing reasoning that may in fairness be rejected by people of sound understanding. (b) A conception of set may be delineated which accords with natural expectations and by means of which we may build a reliable theory of sets comprehending virtually all the results that have been subjected to attack. The relation is indicated which this conception bears to the basic conceptions, in the matter at issue, of the intuitionists (Brouwer and his school), the formalists (Hilbert and his school), the logicians (Russell and others), and the postulationists (Zermelo, Fraenkel). (Received July 29, 1941.)

430. A. R. Schweitzer: *Concerning general abstract relational spaces. III.*

On the basis of the general abstract relational space $S_{n+1}(G, H)$ $(n=1, 2, 3, \cdots)$, $G=H=$symmetric group on $n+1$ variables and certain axioms elaborating this space, the author constructs axioms for the (finite) algebra of logic analogous to his system ${}^{n+1}K_{n+1}$ for the foundations of geometry. For $n=3$ the elements of S_4 are α, β, γ, δ, λ, μ, ν, ω with $\alpha\beta\gamma\delta\ K$, that is, $\alpha\beta\gamma\delta\ K\ \alpha\beta\gamma\delta$. Axioms elaborating S_4 are the following: 1. $\alpha\beta\gamma\delta\ K \supset \lambda\mu\nu\omega\ K$. 2. $\alpha\beta\gamma\delta\ K$ and $\xi \supset \xi\beta\gamma\delta\ K$ or $\alpha\xi\gamma\delta\ K$ or $\alpha\beta\xi\delta\ K$ or $\alpha\beta\gamma\xi\ K$. 3. $\alpha\beta\gamma\delta\ K$ and $\lambda\mu\nu\omega\ K$ and ξ, $\eta \supset \alpha\lambda\xi\eta$ not K, $\beta\mu\xi\eta$ not K, $\gamma\nu\xi\eta$ not K, $\delta\omega\xi\eta$ not K. The complete set of 2^4 K tetrads is expressed as a reflexive formal sum $\sum(I)$ and classified into subsums: $\sum(\xi)$ is the sum of all K tetrads containing ξ, and so on. If $\xi\eta\zeta\tau\ K$, then $\sum(\xi\eta\zeta\tau)=\xi\eta\zeta\tau$. The existence of a unique "empty" sum $\sum(0)=\sum(\alpha\lambda)=\sum(\beta\mu)=\sum(\gamma\nu)=\sum(\delta\omega)$ is assumed. The summands of the various $\sum$'s are replaced by their corresponding expressions in terms of $\sum$ and the $\sum$'s are then represented as products $\sum(\xi\eta)=\sum(\xi)\times\sum(\eta)$, and so on. The preceding continues a paper reported in this Bulletin (abstract 46-9-438). (Received July 22, 1941.)

Statistics and Probability

431. K. J. Arnold: *On spherical probability distributions.*

Two methods of correspondence for circular distributions to the normal error function have led to non-constant absolutely continuous functions (see F. Zernike, *Handbuch der Physik*, vol. 3, pp. 477–478). The corresponding distributions for the sphere are found. The case of diametrical symmetry for both circle and sphere is discussed. Tables of the probability integrals involved are given and an application in geology is included. (Received July 31, 1941.)

432. I. W. Burr: *Cumulative frequency functions.*

Frequency and probability functions play a fundamental role in statistical theory and practice. They are, however, often inconvenient and difficult to use, since it is necessary to integrate or sum to find the probability for a given range. Theoretically the cumulative or integral frequency function would seem to be better adapted to determining such probabilities, since the latter can be found simply by a subtraction. The aim of this paper is to make a contribution toward the direct use of cumulative frequency functions. Some general properties and theory of cumulative functions are presented with particular emphasis upon certain moment functions adapted to such direct use. Both continuous and discrete cases are included. A list of possible cumulative functions is given and a particular one, $F(x)=1-(1+x^c)^{-k-1}$, discussed fully. This function has properties which make it practicable and adaptable to a wide variety of distribution types. It well illustrates the possibilities of the cumulative approach. (Received July 8, 1941.)

433. J. B. Coleman: *A geometric derivation of Fisher's z-transformation.*

In fitting points in a plane by a line so that the sum of the squares of the perpendicular deviations shall be a minimum, a second line is found for which the sum of the squares of the deviations is a maximum. Let $\sum d^2$ be the sum of the squares of the deviations of the points from the minimum line, and $\sum D^2$ be the sum of the squares from the maximum line. Then $\sum D^2/\sum d^2 = [(1+r)/(1-r)]$, and $\frac{1}{2}\log(1+r)/(1-r)$ is Fisher's z-transformation for testing the coefficient of correlation. (Received July 21, 1941.)

434. J. H. Curtiss: *On the distribution of the quotient of two chance variables.*

The purpose of this paper is to give an accurate general treatment of the distributions of the quotient and product of two chance variables, with attention first to questions of existence, and then to the derivation of a number of formulas for the frequency functions and distribution functions. The principal formulas derived are (i) a formula for the frequency function of the quotient of two variables with an absolutely continuous joint probability function, (ii) a formula for the distribution function of the quotient of a pair of arbitrary independent variables expressed in terms of the distribution functions of these variables, (iii) a formula similar to (ii), but expressed in terms of the characteristic functions of the variables. Variable distributions are also considered, and a theorem for quotients analogous to the central limit theorem is derived. (Received July 8, 1941.)

435. W. K. Feller: *On the integral equation of the renewal theory.*

As is well known, the equation $U(t)=G(t)+\int_0^t U(t-x)dF(x)$ has frequently been discussed under different forms in connection with the population theory, the theory of industrial replacement, and so on. In the present paper it is shown that, using Tauberian theorems for Laplace integrals, it becomes possible to analyze in detail the asymptotic behavior of $U(t)$ as $t\to\infty$, and also to solve some other problems which have been discussed in the literature. Strict conditions for the validity of different methods to treat the equation are given together with some modifications found to be necessary. The paper will appear in the Annals of Mathematical Statistics. (Received July 30, 1941.)

436. A. M. Mood: *On the asymptotic distribution of medians of samples from a multivariate population.*

Let two variates x_1 and x_2 have a density function $f(x_1, x_2)$ which, besides being positive or zero and having its integral over the whole space equal to one, shall satisfy these conditions: $\int_{-\infty}^{\infty} f(x_1, 1/n)dx_1 = \int_{-\infty}^{\infty} f(x_1, 0)dx_1 + O(1/n)$, $\int_{-\infty}^{\infty} f(1/n, x_2)dx_2 = \int_{-\infty}^{\infty} f(0, x_2)dx_2 + O(1/n)$. The coordinate system is assumed to have been chosen so that the population median is at the origin. Let $(\bar{x}_1, \bar{x}_2)$ be the median of a sample of $2n+1$ elements drawn from a population with this density function. It is shown that for large samples $(\bar{x}_1, \bar{x}_2)$ is normally distributed to within terms of order $n^{-1/2}$ with zero means and variances and covariances given by certain integrals of $f(x_1, x_2)$. A similar result is true for k as well as two variates. (Received August 2, 1941.)