

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

293. A. H. Clifford: *Matrix representations of completely simple semigroups.*

By a representation \mathfrak{T}^* of degree n of a semigroup S is meant a mapping $a \rightarrow T^*(a)$ of S into the set of all square matrices of n rows and columns such that $T^*(ab) = T^*(a)T^*(b)$. A. Suschkewitsch (Communications de la Société Mathématique de Kharkow, vol. 6 (1933), pp. 27–38) partially solved the problem of determining all representations of a type of semigroup which he calls a “Kerngruppe.” In the terminology of D. Rees (Proceedings of the Cambridge Philosophical Society, vol. 36 (1940), pp. 387–400), the latter is a “completely simple semigroup without zero.” The present paper completes the solution, at the same time allowing S to be any completely simple semigroup. To any such S belongs a “structure group” G , and every representation \mathfrak{T}^* of S is an “extension” of a representation \mathfrak{T} of G . To any given \mathfrak{T} corresponds a certain matrix H , possibly infinite. \mathfrak{T} admits an extension \mathfrak{T}^* of finite degree if and only if H has finite rank h . If \mathfrak{T} has degree n , there exists a “basic” extension \mathfrak{T}_0^* of degree $n+h$, and every extension \mathfrak{T}^* of \mathfrak{T} , even though it may be indecomposable, reduces into \mathfrak{T}_0^* and null representations. (Received March 31, 1941.)

294. C. J. Everett: *Vector spaces over rings.*

Every basis of a vector space V_n (with n basis elements) over a ring K with right ideal maximal condition has n elements. A ring is exhibited over which any V_n is always a V_1 . Every submodule M of a V_n over ring K has a basis of $b(M) \leq n$ elements if and only if (property P) every right ideal of K is principal of type rK , where r is a non-left-zero-divisor. For V_n over K with property P , $b(M)$ is a positive modular functional (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, p. 40). If K has property P and is without zero-divisors, the metric homomorph (loc. cit., Theorem 3.10) of the lattice of submodules of V_n over K is lattice-isomorphic with the lattice of submodules of V_n over the quotient field of K . For V_n over K of type P , $b(M)$ is sharply positive (loc. cit., p. 41) if and only if K is a quasi-field. (Received May 26, 1941.)

295. A. W. Jones: *On the characterization of groups whose lattices satisfy specified lattice identities.*

It is shown that the lattice of a group is simple, complemented, and modular (that is, capable of abstractly representing a finite projective geometry) if and only if the group is either: (1) abelian of prime power order and with prime order elements

only, or (2) a non-abelian group of order pq^n generated by the elements a and b_i ($i=1, 2, \dots, n$) which satisfy the relations $a^p = b_i^q = 1$ and $a^{-1}b_i a = b_i^{u_i}$ (p divides $q-1$ and q divides $u^p - 1 \neq 0$). A group lattice is complemented and modular if and only if the group is the direct product of groups of relatively prime orders and of types (1) or (2) above. (Received May 21, 1941.)

296. Leonard Carlitz: *An analogue of the Bernoulli polynomials.*

The polynomials in question are defined by means of $\psi(tu)/u\psi(t) = \sum \beta_m(u)t^m/g_m$, which may be compared with $t/\psi(t) = \sum B_m t^m/g_m$. (For the quantities involved see *An analog of the Staudt-Clausen theorem*, Duke Mathematical Journal, vol. 3 (1937), p. 503; also, vol. 7 (1940), p. 62.) If u is put equal to U , a polynomial in x over $GF(p^n)$, then $\beta_m(U)$ becomes a rational fraction in x alone. This paper contains various theorems of an arithmetic nature on $\beta_m(U)$, in particular a theorem of the Staudt-Clausen type on the fractional part. (Received April 11, 1941.)

297. Mark Kac: *Remark on the distribution of values of the arithmetic function $d(m)$.*

Let $d(m)$ denote the number of different divisors of m (1 and m included) and $r_n(\omega)$ the number of positive integers less than or equal to n for which $d(m) \leq 2 \log \log n + \omega(2 \log \log n)^{1/2}$. Then $\lim_{n \rightarrow \infty} r_n(\omega)/n = \pi^{-1/2} \int_{-\infty}^{\omega} e^{-u^2} du$. (Received May 2, 1941.)

298. Joseph Lehner: *A partition function connected with the modulus five.*

Let $p_1(n)$, $p_2(n)$ denote the number of partitions of a positive integer n into summands of the form $5l \pm 1$ and $5l \pm 2$, respectively. These functions occur in identities of I. Schur, in which they are related to the number of partitions of n into summands which differ by at least two. Convergent series for $p_1(n)$ and $p_2(n)$ are derived by using the Hardy-Littlewood method in the strengthened form of Kloosterman-Rademacher. The transformation equations of the generating functions of $p_1(n)$, $p_2(n)$, which are closely related to certain modular forms of dimension zero, yield new relations in the theory of theta-functions. The familiar exponential sums which arise from the application of the Hardy-Littlewood method are reduced by a number-theoretic method to Kloosterman sums, thereby permitting the sharper estimate $O(n^{1/3} k^{2/3+\epsilon})$ which is needed for the application of Rademacher's method to modular forms of dimension zero. (Received April 2, 1941.)

299. J. B. Rosser: *A generalization of the euclidean algorithm to several dimensions.*

If $Y = \sum a_i X_i$, the a 's being integers, Y is said to be an I.L.C. (integral linear combination) of the X 's. A set of vectors U_1, \dots, U_r is called a G.C.F. of a set of vectors V_1, \dots, V_s if: I. The U 's are linearly independent. II. Each U is an I.L.C. of the V 's. III. Each V is an I.L.C. of the U 's. A set of V 's is said to be commensurable if the set of I.L.C.'s of the V 's has no limit point. It is proved that every commensurable set of vectors has a G.C.F. The G.C.F. of two commensurable collinear vectors is a single vector, which can be found by the euclidean algorithm. Generalizing the euclidean algorithm in a rather obvious fashion, one has a procedure for finding a G.C.F. of more general sets of vectors. This algorithm solves numerous problems

relative to positive definite quadratic forms and Diophantine equations. For two incommensurable collinear vectors, the euclidean algorithm becomes the continued fraction algorithm, a powerful tool in approximation problems. It is conjectured that the generalized algorithm is as effective for problems of simultaneous approximation as the continued fraction algorithm is for simple approximation problems. To support this conjecture, satisfactory solutions of such problems are obtained by use of the generalized algorithm. (Received April 24, 1941.)

300. Albert Whiteman: *Sums connected with the partition function.*

The sum $A_k(n) = \sum \exp(\pi i s(h, k) - 2\pi i h n/k)$, where h runs over a reduced residue system with respect to the modulus k and $s(h, k)$ is a Dedekind sum, appears in Rademacher's formula for the number of partitions of n . On the basis of a different expression for this sum Lehmer (Transactions of this Society, vol. 43 (1938), pp. 271-295) factored the $A_k(n)$ according to the prime number powers contained in k , and evaluated the $A_k(n)$ in the case in which k is a prime or a power of a prime. A new approach to the first of these results has recently been given in a paper by Rademacher and Whiteman (American Journal of Mathematics, vol. 63 (1941), pp. 377-407). In the present paper the second of these results is derived by a method which is considerably simpler than Lehmer's. The paper also contains a new method for evaluating certain generalized Kloosterman sums. (Received May 29, 1941.)

ANALYSIS

301. R. P. Agnew: *On limits of integrals.*

It is shown that existence of $\lim_{A \rightarrow \infty} \int_A^{A+\lambda} f(t) dt$, for each λ in some set having positive measure, implies that the limit exists and is uniform over each finite interval of values of λ . The result is applied to prove two theorems of Iyengar (Proceedings of the Cambridge Philosophical Society, vol. 37 (1941), pp. 9-13) and the following Tauberian theorem. If $F(t)$ is absolutely continuous over each finite interval, if $\lim_{A \rightarrow \infty} \int_A^{A+\lambda} [F(t) - F'(t)] dt = 0$ for each real λ , and if $\lim_{t \rightarrow \infty} e^{-t} F(t) = 0$, then $\lim_{t \rightarrow \infty} F(t) = 0$. (Received April 11, 1941.)

302. Stefan Bergman: *A method for summation of series of orthogonal functions of two variables.*

Suppose $\{\Phi_\nu(\phi)\}$ [$(\phi) = (\phi_1, \phi_2)$] is a system of O.N. functions $[0 \leq \phi_k \leq 2\pi, \Phi_k(\phi_1 + 2\pi, \phi_2) = \Phi_k(\phi_1, \phi_2 + 2\pi) = \Phi_k(\phi_1, \phi_2), k = 1, 2]$, $\sigma(e)$ a completely additive set function, $f(\phi_1, \phi_2) \in L^{1+p}$ ($p > 0$), $a_\nu = \int_0^{2\pi} \int_0^{2\pi} \Phi_\nu d\sigma$, $b_\nu = \int_0^{2\pi} \int_0^{2\pi} \Phi_\nu f d\phi_1 d\phi_2$. Consider the series (1) $\sum_{\nu=0}^{\infty} a_\nu \Phi_\nu(\phi_1, \phi_2)$ and (2) $\sum_{\nu=0}^{\infty} b_\nu \Phi_\nu(\phi_1, \phi_2)$. Let \mathfrak{M} be a four-dimensional domain of the type described in Mathematische Annalen, vol. 104 (1931), pp. 611-636, with the distinguished boundary surface $\mathfrak{F} = E[z_k = h_k(\phi_1, \phi_2), k = 1, 2]$. Let $\Psi_\nu(z_1, z_2), \nu = 1, 2, \dots$, be the functions of the extended class which assume the values Φ_ν on \mathfrak{F} , and (3) $S(z_1, z_2) = \sum_{\nu=0}^{\infty} a_\nu \Psi_\nu(z_1, z_2)$, (4) $F(z_1, z_2) = \sum_{\nu=0}^{\infty} b_\nu \Psi_\nu(z_1, z_2)$. The series (4) converges absolutely and uniformly in every closed subdomain of \mathfrak{M} for $p > 1$. Using the results of Jessen, Marcinkiewicz and Zygmund (Fundamenta Mathematicae, vol. 25 (1935), pp. 217-234), Bergman and Marcinkiewicz (Fundamenta Mathematicae, vol. 33 (1939), pp. 75-94) and Bers (Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), pp. 1273-1275 and 1475-1477) it is shown that S possesses a finite sectorial limit at the point $\{h_1(\phi), h_2(\phi)\}$ if σ possesses a finite strong derivative at (ϕ) , and F possesses the sectorial limit f almost everywhere on \mathfrak{F} . (Received May 2, 1941.)