choose a critical region $W_n$ of the $n$-dimensional sample space and reject the hypothesis $H$ if $E=(x_1, \cdots, x_n)$ falls inside $W_n$. Denote by $P(W_n|\theta_1, \cdots, \theta_k)$ the probability that $E$ will fall in $W_n$ under the assumption that $\theta_1, \cdots, \theta_k$ are the true values of the parameters and denote by $P_n(\theta_1, \cdots, \theta_k, \alpha)$ the least upper bound of $P(Z_n|\theta_1, \cdots, \theta_k)$ with respect to all regions $Z_n$ for which $P(Z_n|\theta_1', \cdots, \theta_k')=\alpha$. A critical region $W_n$ is called a most stringent test of the hypothesis $H$ on the level of significance $\alpha$ if $P(W_n|\theta_1', \cdots, \theta_k')=\alpha$ and l.u.b. $\{P_n(\theta_1, \cdots, \theta_k, \alpha)-P(W_n|\theta_1, \cdots, \theta_k)\}$ $\leq$l.u.b. $\{P_n(\theta_1, \cdots, \theta_k, \alpha)-P(Z_n|\theta_1, \cdots, \theta_k)\}$ (l.u.b. with respect to $\theta_1, \cdots, \theta_k$) for any region $Z_n$ for which $P(Z_n|\theta_1', \cdots, \theta_k')=\alpha$. It is shown that the test of $H$ based on the so-called likelihood ratio introduced by Neyman and Pearson is a most stringent test in the limit if $n\rightarrow\infty$. The foregoing definitions and results are extended also to testing composite hypotheses. (Received February 3, 1941.)

## Topology

### 276. G. E. Albert: *On separation spaces.* Preliminary report.

A. D. Wallace has introduced separation spaces (abstract 46-7-368), adopting as a primitive concept a binary relation $X|Y$, between pairs of non-vacuous subsets $X$ and $Y$ of an abstract set $S$. Subject to certain axioms, $X|Y$ can be used to define a topology in $S$ which makes the resulting space completely equivalent to a $T_1$-topological space. A prominent part is played by the axiom: $X|Y$ implies $Y|X$. In the present paper separation spaces are studied in which this property of symmetry is discarded. It is shown that, subject to proper alterations of Wallace's axioms, separation spaces can be used to characterize $T_0$-topological spaces. The theory of asymmetrical separation is found to be particularly convenient for the definition of a topology in upper semi-continuous collections of type 2 (see R. L. Moore, Rice Institute Pamphlets, vol. 23, no. 1). Other applications of a more general nature are also indicated. (Received March 10, 1941.)

### 277. G. E. Albert and J. W. T. Youngs: *The structure of locally connected topological spaces.*

The present paper is a continuation of earlier work (abstract 46-3-138). Cyclic elements are defined and studied in the class $L$ of spaces which satisfy the postulates: the space and the empty set are open; the intersection (sum) of finitely (arbitrarily) many open sets is open; the components of an open set are open. The hyperspace $\mathfrak{X}$ of all cyclic elements of any space $X\in L$ is topologized in such a way that: (1) $\mathfrak{X}$ is a space $L$, (2) $\mathfrak{X}$ is a strongly continuous image of $X$, and (3) $\mathfrak{X}$ is acyclic. A subclass $H$ of $L$ is called hereditary if $X\in H$ implies that the hyperspace of $X$ is in $H$ and every true cyclic element of $X$ is a space in $H$. The class $L$ is a hereditary class which contains the class $P$ of all Peano spaces; however, it deviates widely from $P$. Hereditary subclasses of $L$ are proposed which approximate more closely the class $P$. For example: one such class is composed of all spaces $L$ which satisfy the $T_0$-separation axiom and which are strongly continuous images of the unit line interval. (Received March 10, 1941.)

### 278. Samuel Eilenberg: *Banach space methods in topology.* I.

For a given topological space $X$ the Banach space $\mathfrak{X}$ of all continuous bounded real functions $\mathfrak{x}$ on $X$ with norm l.u.b. $|\mathfrak{x}(x)|$ is considered. Banach has proved that two compact metric spaces, $X_1$ and $X_2$, are homeomorphic if and only if $\mathfrak{X}_1$ and $\mathfrak{X}_2$ are isometric. Stone (Transactions of this Society, vol. 41 (1937), p. 469) has generalized this theorem to normal bicompact spaces $X$. The author obtains the same result for normal compact spaces satisfying the first countability axiom. The method consists in an analysis of the convex subsets of the surface of the unit sphere in $\mathfrak{X}$. A complete reconstruction of $X$, under either Stone's or the author's hypotheses, may then be obtained. (Received March 12, 1941.)

### 279. Samuel Eilenberg: *Banach space methods in topology*. II.

A Banach space $B$ is a direct product, $B_1 \times B_2$, of two of its closed linear subspaces $B_1$ and $B_2$, if every element $b \in B$ has a unique decomposition $b = b_1 + b_2$, $b_i \in B_i$, and if $\|b\| = \max (\|b_1\|, \|b_2\|)$. Given a topological space $X$ decomposed into two disjoint closed sets $X_1$ and $X_2$, the Banach space $\mathfrak{X}$ (see abstract 47-5-278) is a direct product $\mathfrak{X}_1 \times \mathfrak{X}_2$ if $\mathfrak{X}_i$ is that subset of $\mathfrak{X}$ that consists of all functions $\mathfrak{x}$ which vanish outside $\mathfrak{X}_i$. If $X$ is normal, compact, and satisfies the first countability axiom, then the converse can be proved, that is, the above direct product decomposition is the most general one. In particular $X$ is connected if and only if $\mathfrak{X}$ is indecomposable. (Received March 12, 1941.)

### 280. Samuel Eilenberg: *Irreducible transformations onto manifolds*.

A continuous mapping $f(X) = M$ of a metric compact space onto an $n$-dimensional manifold is called irreducible (equal to strongly irreducible in the terminology of G. T. Whyburn, American Journal of Mathematics, vol. 61 (1939), p. 820) if $M - f(A) \neq 0$ for every closed proper subset $A$ of $X$. A point $y \in M$ is said to be covered essentially by $f$ if there is a neighborhood $U$ of $y$ such that there is no mapping $g(X) \subset M - y$ such that: $g(x) = f(x)$ if $f(x) \in M - U$, $g(x) \in U$ if $f(x) \in U$. The following is proved: if $f$ is irreducible and $y \in M$ is covered essentially, then $f^{-1}(y)$ is a continuum. In particular if $f$ is irreducible and essential in the sense of Hopf, then $f$ is monotone. (Received March 12, 1941.)

### 281. Harlan C. Miller: *Concerning compact unicoherent continua*.

In this paper it is proved that in order that the compact hereditarily decomposable continuum $M$ be hereditarily unicoherent it is necessary and sufficient that $M$ contain no continuum $N$ such that $N$ is a simple closed curve with respect to the elements of an upper semicontinuous collection of mutually exclusive continua filling up $N$. The author has previously proved that in order that the compact continuum $M$ be atriodic and hereditarily unicoherent it is necessary and sufficient that of any three points of $M$ there is one which weakly separates the other two from each other in $M$. It is now shown that (1) if $M$ is a compact hereditarily decomposable continuum and there exists a positive integer $k$ such that of any $k$ points of $M$ there is one which weakly separates two of the others from each other in $M$, then $M$ is hereditarily unicoherent, and that the stipulation that $M$ be hereditarily decomposable can be omitted only if $k < 5$, and (2) if $M$ is a compact hereditarily decomposable continuum such that every subcontinuum of $M$ is irreducible about some (closed) proper subset having only countably many components, then $M$ is hereditarily unicoherent. (Received March 13, 1941.)

### 282. W. T. Puckett: *Concerning a transformation in the plane*. Preliminary report.

The single-valued continuous transformation $T(P) = P'$ will be called *quasi 0-regu-*

*lar* provided that (1) $T(P-Q)$ is interior for some discrete set $Q$ and (2) for each $p \in P-Q$ and neighborhood $U(p)$ there exists a neighborhood $V(p)$ such that $(V(p)-Q)\cdot T^{-1}(x')$ lies in a single component of $(U(p)-Q)\cdot T^{-1}(x')$ for any $x' \in P'$. Under the hypotheses that $P$ is the plane and each $T^{-1}(x')$ has only nondegenerate components it is shown that (1) every point of $P'$ is a local separating point of order 2 and (2) any $T^{-1}(x')$ may be characterized as a discrete collection of open arcs, a finite number of which may have a point $q$ of $Q$ as a common end point. Moreover, the behavior of the transformation near such a point $q$ is exhibited. This theory will be applied to a topological characterization of potential functions. (Received March 10, 1941.)

### 283. G. E. Schweigert: *Border transformations.* II.

If $A$ is compact metric and if the light factor of the monotone light factorization of a continuous transformation, $T(A)=B$, is interior, then $T$ is said to be quasi-monotone. This new definition (A. D. Wallace, Duke Mathematical Journal, vol. 7 (1940)) gives a desirable property which, an example shows, does not hold under the old except for special spaces. Quasi-monotone is characterized by the statement: a sequence converging to $y \in B$ implies that the limit superior of the counterimages of the sequence intersects each component of $T^{-1}(y)$. This permits comparison with the border transformation where the limit of the counterimages is the border of $T^{-1}(y)$. Also, if $I(M)$ denotes the interior of $M$, the necessary condition $T^{-1}(I(Y))+\sum I(T^{-1}(y))=I(T^{-1}(Y))$, where the summation is over the border of $Y$, is shown to be sufficient that $T$ be a border transformation. Two other necessary conditions are shown to be sufficient. (Received March 13, 1941.)

### 284. M. E. Shanks: *The space of all metrics on a compact space.*

If $X$ is a compact metrizable space, the space $M(X)$ of all metrics on $X$, which preserve the topology, and their limits is a complete, separable positively-linear normed space. $M(X)$ is unique, that is, $M(X)$ is congruent to $M(Y)$ if and only if $X$ is homeomorphic to $Y$. $M(X)$ contains in a sense all continuous transformations of $X$. If $M^*(X)$ denotes the linear extension of $M(X)$, then $M^*(X)$ is a universal metric space if $X$ has the power of the continuum. In fact it is possible to define a whole class of universal metric spaces by taking different classes of metrices on $X$. The methods are largely those of Banach. (Received March 14, 1941.)

### 285. N. E. Steenrod and A. W. Tucker: *Real n-quadrics as sphere-bundles.*

By a suitable choice of homogeneous coordinates in real projective $(n+1)$-space the equation of a real $n$-quadric $Q$ can be written $x_0^2+x_1^2+\cdots+x_k^2=x_{k+1}^2+\cdots+x_{n+1}^2$ $(k \leqq n-k)$. The mapping $(x_0:x_1:\cdots:x_{n+1}) \rightarrow (x_0:x_1:\cdots:x_k:0:\cdots:0)$ sections the quadric $Q$ into a continuous collection of $(n-k)$-spheres, one for each point of projective $k$-space. This collection is a sphere-bundle in the sense of Whitney (Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 148–153). Using results of Radon on linear sets of orthogonal matrices (Abhandlungen des mathematischen Seminars der Hamburgischen Universität, vol. 1 (1921), pp. 1–14) it can be shown that $Q$ is topologically equivalent to the product $P \times S$ of projective $k$-space $P$ and an $(n-k)$-sphere $S$ provided $n-k$ is odd and $k$ is small enough (if $\lambda$ denotes the exponent of the highest power of 2 which divides $n-k+1$ then, according as $\lambda \equiv 0, 1, 2, 3 \bmod 4$,

$k<2\lambda+1$, $2\lambda$, $2\lambda$, $2\lambda+2$, respectively). For $n-k$ odd and $k$ larger it seems probable that $Q$ and $P\times S$ are not topologically equivalent, although they have isomorphic homology groups. For $n-k$ even and $k>0$ the quadric $Q$ and the product $P\times S$ differ in orientability but have isomorphic homology groups mod 2. (Received March 14, 1941.)

### 286. A. W. Tucker: *Simplicial bands.*

Consider a simplicial complex formed by a circular sequence of $k$ $n$-simplexes, each of which has an $n-1$-face in common with the next ($k\geqq 2n+1$, necessarily). This *simplicial band* has as many $p$-simplexes as $n-p$-simplexes and its boundary is a regular $n-1$-polyhedron. If $n$ and $k$ are both even or if $n$ is odd, it is the topological product of a circle and a closed $n-1$-cell. If $n$ is even and $k$ odd, it is an $n$-dimensional analogue of a Möbius band, obtained from an $n$-cylinder by a sense-reversing identification of the base and cap. If $n$ or $k$ is odd, it is topologically equivalent to each of the following: (1) the symmetric product of $n$ circles; (2) the closed region of real projective $n$-space which represents the subset of real equations of degree less than or equal to $n$ with real roots, when coefficients are used as homogeneous coordinates; and (3) the quadric-bounded projective region $x_0^2+x_1^2\geqq x_2^2+\cdots+x_n^2$. (Received March 8, 1941.)

### 287. A. D. Wallace: *A fixed-point theorem for trees.*

Let $T$ be a compact Hausdorff space which is locally connected and which is acyclic in the sense that every finite open covering admits a finite open refinement whose nerve is a tree. Let $q$ be a function which assigns to each point $t$ in $T$ a continuum $qt$ in $T$, and which is continuous in the sense that if $t$ and a neighborhood $U$ of $qt$ are given then there is a neighborhood $V$ of $t$ such that if $t'$ is in $V$ then $qt'$ is in $U$. In this note it is shown that *there is a* $t_0$ *in* $T$ *such that* $t_0$ *is in* $qt_0$. This is a generalization of the Scherrer fixed-point theorem for acyclic continuous curves and is analogous to a recent result of Kakutani. As a corollary to our result we deduce a known result on the nonexistence of free monotone transformations into trees and a new theorem on the existence of coincidences of monotone transformations into trees. (Received March 7, 1941.)

### 288. P. A. White: *A decomposition of true cyclic elements by means of continua.*

In this paper a decomposition of a cyclicly connected locally connected continuum is defined by dividing the continuum into certain sets which have the property that no two points in them can be separated by a continuum. These sets are divided into two types according as they do or do not contain points of Menger order two. It is shown that each set of the first type is contained in a simple closed curve and that they are at most countable in number, while each set of the second type is at most countable. Other results are given concerning the components of these sets and of their complements. (Received March 31, 1941.)

### 289. G. W. Whitehead: *Homotopy properties of rotation groups.*

This paper continues the investigation of homotopy properties of the rotation groups $R_n$ of $n$-spheres begun by Hurewicz and Steenrod (Proceedings of the National

Academy of Sciences, vol. 27 (1941), pp. 61–64). The homotopy groups $\Pi_i(R_n)$ are computed for $i=3$, 4, 5 and all $n$, with the following results: $\Pi_3(R_n)$ is the infinite cyclic group ($n=2$, $n\geqq 4$); $\Pi_3(R_3)$ is the direct sum of two such groups; $\Pi_4(R_2)=\Pi_4(R_4)$ is the cyclic group of period 2, $\Pi_4(R_3)$ is the direct sum of two such groups, $\Pi_4(R_n)=0$ ($n\geqq 5$); $\Pi_5(R_n)=0$ ($n\neq 5$), and $\Pi_5(R_5)$ is the infinite cyclic group. It is proved that if $n>1$ and $n\not\equiv 3$ (mod 4), there exists no map $f$ of $S^n$ into $R_n$ such that the projection $\Pi f$ into $S^n$ has odd degree. Thus for these values of $n$ there do not exist $n$ independent fields of tangent vectors on $S^n$, and the tangent sphere-bundle is not simple. In fact any two vector fields are somewhere dependent. (Received March 13, 1941.)

## 290. G. T. Whyburn: *Almost periodicity.*

Let $f(X)=X$ be a homeomorphism where $X$ is a metric space. It is shown that $f$ is pointwise periodic if and only if for any subset $Y$ of $X$, $f(Y)\subset Y$ implies $f(Y)=Y$ and that, for compact sets $X$, $f$ is pointwise almost periodic (see Ayres, Fundamenta Mathematicae, vol. 33, reprint) if and only if $f(K)\subset K$ implies $f(K)=K$ for every compact subset $K$ of $X$. A lemma is then proven to the effect that if $X$ is a continuum and $X=H+K$ is any division of $X$ into continua $H$ and $K$ such that $H\cdot K$ is a single point $p$ and $H\cdot f(H)\neq 0\neq K\cdot f(K)$, then pointwise almost periodicity of $f$ at $p$ implies $f(p)=p$. On the basis of this characterization and lemma, a considerably simplified development (using nowhere the notion of linear order) is given of the results of Ayres on pointwise almost periodic homeomorphisms—at the same time extending them to semi-locally-connected continua. (Received March 12, 1941.)

## 291. G. T. Whyburn: *Orbit decompositions.*

Let $f(X)\subset X$ be continuous where $X$ is metric. For any $x\in X$, the set $A_x$ of all $y\in X$ such that $f^m(x)=f^n(y)$ for positive integers $m$ and $n$ will be called the *orbit* of $x$. A subset $A$ of $X$ is *completely invariant* provided $f(A)=A$ and $f^{-1}(A)=A$. The set $A_x$ then turns out to be the smallest completely invariant subset of $X$ containing $x$. The decomposition of $X$ into the (disjoint) sets $[A_x]$ is called the *orbit decomposition*. It may be obtained also by means of the equivalence relation: $x\equiv y$ provided $f^m(x)=f^n(y)$ for some $m$, $n>0$. Assuming $X$ compact and the orbits closed, it is found that the orbit decomposition is upper semi-continuous provided $f$ has *equi-continuous powers*, that is, for any $x\in X$ and $\epsilon>0$ a neighborhood $U$ of $x$ exists so that $\rho[f^n(x), f^n(y)]<\epsilon$ for all $y\in U$ and all $n>0$. If $f$ is pointwise periodic, the converse holds and further, in this case, the condition is equivalent to *regular almost periodicity* of $f$, that is, for any $\epsilon>0$ an $n$ exists so that $\rho[x, f^{mn}(x)]<\epsilon$ for all $x\in X$ and all $m>0$. Also, interiority of $f$ implies pointwise periodicity, and the orbit decomposition is actually continuous. (Received March 12, 1941.)

## 292. G. T. Whyburn: *Regular almost periodicity.*

A homeomorphism $f(X)=X$ is regularly almost periodic if for each $\epsilon>0$ an integer $n$ exists such that $\rho[x, f^{mn}(x)]<\epsilon$ for all $x\in X$ and all integers $m>0$. Regular almost periodicity is cyclicly extensible in semi-locally-connected continua $X$. If each cyclic element of such an $X$ is locally euclidean, any pointwise periodic mapping on the non-end-points of $X$ is regularly almost periodic on $X$. A simplified treatment of the results of Hall and Schweigert (Duke Mathematical Journal, vol. 4 (1938), p. 719; this Bulletin, vol. 46 (1940), p. 963) is given making no use of transfinite ordinals or induction. It is shown that any pointwise periodic mapping $f(X)\subset X$, $X$ compact, is monotonically equivalent to a pointwise periodic and regularly almost periodic mapping $g(X')=X'$, that is, there exists a monotone mapping $\phi(X)=X'$ such that $g(x')=\phi f\phi^{-1}(x')$, $x'\in X'$. The author obtains also a new proof of the 2-dimensional case of a theorem of Montgomery (American Journal of Mathematics, vol. 59 (1937), p. 118) based on the fact that a finite-to-one interior mapping of any 2-manifold (compact or not) into a connected set has a finite degree $k$ and the set of points of multiplicity less than $k$ is one-dimensional, rather than on Newman's theorem. (Received March 25, 1941.)