another line L. Any such set may be considered to be analogous to the velocity systems as developed by Kasner. Any line transformation, not preserving all parallel pencils of lines, converts exactly one dual-velocity system into a dual-velocity system. The group preserving all dual-velocity systems is $X=\phi(x)$, $Y=y\psi(x)+\chi(x)$. This is the contact group leaving invariant the set of all dual-isothermal families. Examples of dual-velocity systems are equitangential, dual-natural, Δ, and dual-Γ families. Characterizations of these are obtained by the correspondence between the lines l and L mentioned above. Any dual-velocity system contains exactly ∞^2, ∞^1, one, or zero dual-isothermal families. Finally, the invariant theory of dual-velocity systems under both the dual-isothermal and equilong groups is developed. A dual-analogue of natural family has been discussed in an earlier paper. (Received February 11, 1941.)

266. L. J. Savage: *Distance spaces.*

By a distance space is meant a set M of elements $p, q, \cdots$ over which is defined a real valued function $D(p, q)$ such that $D(p, q)=D(q, p)$, and $D(p, p)=0$. $D(p, q)$ may be thought of as the square of the distance from p to q. This concept is some generalization of metric space. A vector space V over which a scalar product $x\cdot y$ is defined can be considered as a distance space by setting $D(x, y)=(x-y)\cdot(x-y)$. For every distance space M there is a "smallest" scalar product space $V(M)$ in which M is imbeddable. An interesting class of distance spaces is that of differentiable manifolds over which a $D(p, q)$ is so defined as to be suitably differentiable when considered as a function of the coordinates. Because of the remark about scalar product spaces these differentiable distance manifolds can be handled much like differentiable submanifolds of euclidean space. In particular the concepts of regularity, tangent-flat, and first and second fundamental form, can be extended to them. Finally there are theorems connecting the possibility of imbedding such manifolds into euclidean and pseudo-euclidean spaces with certain restrictions on the second fundamental form. (Received March 13, 1941.)

267. R. K. Wakerling: *On the rational loci of ∞^1 $(\rho-1)$-spaces in r-space.*

The representation upon a ρ-space of the hypersurface W_ρ^n in S_r, which is the rational locus of ∞^1 $(\rho-1)$-spaces, is investigated in this paper. The hyperplane sections of W_ρ^n are represented in S_ρ by a system of hypersurfaces $V_{\rho-1}^\nu$ passing through a given $(\rho-2)$-space $\nu-1$ times, and having in common σ simple points. Some of the properties of W_ρ^n are discussed together with those of its projection upon a $(\rho+1)$-space. A special case of the transformation between two r-spaces is given, and the paper is concluded with a brief note on rational ruled surfaces of order $r-1$ in S_r. (Received March 8, 1941.)

Statistics and Probability

268. A. H. Copeland: *If.*

If one attempts to apply Boolean algebra to the theory of probability, he discovers that it is inadequate for the treatment of conditional probabilities, selections, and observations—all three of which are of prime importance in the modern theories of probability and statistics. In a number of recent formalizations of the theory of probability an additional operator (or logical constant) "if" has been introduced in order to handle conditional probabilities. Selections and observations were not introduced into the formal schemes but were merely discussed as interpretations. In a previous paper the author showed that by a proper definition of "if" selections and observations could be formalized. However, this operator was defined in terms of certain extraneous concepts, namely, probabilities and order relations. In the present paper "if" is defined only by its relation to the Boolean operators, "and," "or," and "not." This more natural definition enables us to study the modified Boolean algebra per se and to investigate the structure of and the relations between its elements. (Received March 31, 1941.)

269. G. B. Dantzig: *Variance of the error of a mean computed by grouping.*

To compute the mean of a large number of observations, the range of the data is often divided into equal intervals and the mean is then computed as if each observation fell at the midpoint of the interval containing it. The error introduced by this short cut comes from two sources: (1) the size of the interval h and (2) the amount of off-centerness t, that is, the amount of shift required to bring the nearest mid-point to the origin. Several authors have noted that the average value over all t of the Sheppards' corrected moments (functions of h and the grouped moments) yields the true moments of the distribution. It is suggested here that the value of t should be selected at random, so that it is equally likely to have any value between 0 and h. Then the first grouped moment (now validly considered as a random variable) has for expected value the true first moment, and for variance $\frac{1}{2}h^2 \cdot \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(x)p(y)B_2[(x-y)/h]dxdy$, where $p(x)$ is the density of distribution and $B_2(\theta) = \theta^2 - \theta + 1/6$ is periodic of period one (second Bernoulli polynomial). If the amount of off-centerness is randomly chosen, Sheppards' corrections are always applicable and the error approximable by square root of the variance. (Received March 10, 1941.)

270. G. E. Forsythe: *Cesàro summability of random variables.*

$\{X_k\}$ and $\{Y_k\}$ are sequences of independent real-valued random variables. $\{X_k\}$ is otherwise arbitrary, while $\{Y_k\}$ is a normal family (Lévy) with all $E(Y_k)=0$. $\|a_{nk}\|$ is the Cesàro C_α matrix $(0<\alpha<\infty)$. Using theorems of Feller and Gnedenko, necessary and sufficient conditions are given for the existence of constants $\{d_{nk}\}$ so that both $\sum_{k=0}^{\infty} a_{nk}(X_k-d_{nk})$ and $a_{nk}(X_k-d_{nk})\to 0$ in probability as $n\to\infty$, uniformly in $k\leqq n$. If $\alpha\geqq 1$, d_{nk} may always be chosen independent of n and α. With natural definitions it then follows that for summability of $\{X_k\}$ to 0, the C_α methods get stronger as α increases. This increase in strength is non-trivial for $0<\alpha<1$, but is essentially trivial for $\alpha\geqq 1$. Using another Gnedenko theorem, a necessary and sufficient condition is given for C_α-summability of $\{Y_k\}$ to the Gaussian distribution $G(x)$. From this it is proved: if $0<\alpha<\frac{1}{2}$ or if $\sum_{k=0}^{\infty} E(Y^2)<\infty$, there can be no C_α-summability of $\{Y_k\}$ to $G(x)$; if $\frac{1}{2}\leqq\alpha<\beta$ and $1\leqq\beta$, $C_\alpha\subseteq C_\beta$; if $\frac{1}{2}\leqq\alpha<\beta$, there always exists $\{Y_k\}$ summable-C_α to $G(x)$ but not summable-C_β to $G(x)$, a perhaps surprising result. A close relation is shown between C_α-summability of $\{Y_k\}$ to $G(x)$ and summability of $\{E(Y_k^2)\}$ to $+\infty$ by ordinary Riesz means of order $2\alpha-2$. Nörlund summability to 0 of symmetric variables $\{Y_k\}$ is also treated. (Received March 31, 1941.)

271. J. A. Greenwood: *A theorem on probability assignments to events.*

Consider the mutually exclusive, exhaustive set of variates of a variable x with a probability distribution function $\phi(x)$ defined over $[a, b]$, and to each value of x an associated number $P(x)$, $0 \leqq P(x) \leqq 1$. $P(x)$ may be called a probability assignment to the event x. Definition: $P(x)$ may be said to be a "sufficient" assignment if, for all x, $P(x) \geqq LS\int_{D_x} d\phi(t)$ where $D_x = D_x(t \in [a, b]; P(t) \leqq P(x))$. In order to ensure the existence of this integral and the succeeding one which defines $E[P(x)]$, it is assumed that D_x is a Borel set. The following theorem is then proved: A necessary condition that a probability assignment $P(x)$ to each of the mutually exclusive, exhaustive values of a stochastic variable x be "sufficient" is that $E[P(X)] = LS\int_a^b P(x)d\phi(x) \geqq \frac{1}{2}$. (Received April 1, 1941.)

272. P. C. Hammer: *On fitting linear functions when all variables are subject to error.*

Among the many recent papers on fitting lines or planes in case all variates are subject to error is one by C. F. Roos which points out the lack of invariance of methods developed prior to that time. Among others the methods of Adcock, Pearson, and a generalization of Pearson's method by E. C. Rhodes seem to lack invariance. Assuming a general normal distribution of error and proceeding along classical lines this paper develops an implicit method of fitting linear functions in n variables which includes that of Pearson as a special case and which is invariant under coordinate transformation. A few geometrical interpretations are made and the cases not leading to a unique solution itemized. A comparison of this method with that of Roos is then made. (Received February 10, 1941.)

273. C. T. Hsu: *Two samples from normal bivariate populations.*

Two samples, each being of two variates (x_1, x_2) and (x_1', x_2'), of size n and n' respectively, are supposed to be drawn at random from two independent normal bivariate populations, with the following distributions: (1) $(2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2})^{-1}$ $\exp\{-1/(2(1-\rho^2))((x_1-\xi_1)/\sigma_1)^2 - 2\rho((x_1-\xi_1)/\sigma_1)((x_2-\xi_2)/\sigma_2) + ((x_2-\xi_2)/\sigma_2)^2]\}$, (2) $(2\pi\sigma_1'\sigma_2'(1-\rho'^2)^{1/2})^{-1}$ $\exp\{-1/(2(1-\rho'^2))[((x_1'-\xi_1')/\sigma_1')^2 - 2\rho'((x_1'-\xi_1')/\sigma_2') \cdot((x_2'-\xi_2')/\sigma_2') + (x_2'-\xi_2')/\sigma_2')^2]\}$, where $\xi_1, \xi_2, \sigma_1, \sigma_2, \rho; \xi_1', \xi_2', \sigma_1', \sigma_2', \rho'$ are the unknown parameters of the populations. Two hypotheses are considered concerning the comparison of correlation coefficients, namely, H_1: Assuming $\sigma_1 = \sigma_2$ and $\sigma_1' = \sigma_2'$; to test $\rho = \rho'$. H_2: Assuming $\sigma_1 = \sigma_2$, $\xi_1 = \xi_2$ and $\sigma_1' = \sigma_2'$, $\xi_1' = \xi_2'$; to test $\rho = \rho'$. Appropriate test criteria are derived for each hypothesis. The distribution of certain of the statistics are obtained in the special case where $n = n'$. Incidentally the distribution of the z of S. S. Wilks (Biometrika, vol. 24 (1932), p. 471) for $p-2$ and any values of a_1 and a_2 is studied. (Received April 1, 1941.)

274. Henry Scheffé: *Note on the reduction of χ^2 for fit of a frequency distribution.*

Pearson reduced χ^2 for the fit of an observed to a theoretical frequency distribution by the use of determinant theory and trigonometric identities (Philosophical Magazine, (5), vol. 50 (1900), pp. 160–163). The object of this note is to simplify the reduction by the use of matrix theory. (Received February 15, 1941.)

275. Abraham Wald: *On testing statistical hypotheses concerning several unknown parameters.*

Let $f(x, \theta_1, \cdots, \theta_k)$ be the probability density function of a random variable x involving k unknown parameters. For testing the simple hypothesis $\theta = \theta_1', \cdots, \theta_k = \theta_k'$, called hypothesis H, by means of n independent observations $x_1, \cdots, x_n$ on x we choose a critical region W_n of the n-dimensional sample space and reject the hypothesis H if $E=(x_1, \cdots, x_n)$ falls inside W_n. Denote by $P(W_n|\theta_1, \cdots, \theta_k)$ the probability that E will fall in W_n under the assumption that $\theta_1, \cdots, \theta_k$ are the true values of the parameters and denote by $P_n(\theta_1, \cdots, \theta_k, \alpha)$ the least upper bound of $P(Z_n|\theta_1, \cdots, \theta_k)$ with respect to all regions Z_n for which $P(Z_n|\theta_1', \cdots, \theta_k')=\alpha$. A critical region W_n is called a most stringent test of the hypothesis H on the level of significance α if $P(W_n|\theta_1', \cdots, \theta_k')=\alpha$ and l.u.b. $\{P_n(\theta_1, \cdots, \theta_k, \alpha)-P(W_n|\theta_1, \cdots, \theta_k)\}$ $\leq$l.u.b. $\{P_n(\theta_1, \cdots, \theta_k, \alpha)-P(Z_n|\theta_1, \cdots, \theta_k)\}$ (l.u.b. with respect to $\theta_1, \cdots, \theta_k$) for any region Z_n for which $P(Z_n|\theta_1', \cdots, \theta_k')=\alpha$. It is shown that the test of H based on the so-called likelihood ratio introduced by Neyman and Pearson is a most stringent test in the limit if $n\rightarrow\infty$. The foregoing definitions and results are extended also to testing composite hypotheses. (Received February 3, 1941.)

Topology

276. G. E. Albert: *On separation spaces.* Preliminary report.

A. D. Wallace has introduced separation spaces (abstract 46-7-368), adopting as a primitive concept a binary relation $X|Y$, between pairs of non-vacuous subsets X and Y of an abstract set S. Subject to certain axioms, $X|Y$ can be used to define a topology in S which makes the resulting space completely equivalent to a T_1-topological space. A prominent part is played by the axiom: $X|Y$ implies $Y|X$. In the present paper separation spaces are studied in which this property of symmetry is discarded. It is shown that, subject to proper alterations of Wallace's axioms, separation spaces can be used to characterize T_0-topological spaces. The theory of asymmetrical separation is found to be particularly convenient for the definition of a topology in upper semi-continuous collections of type 2 (see R. L. Moore, Rice Institute Pamphlets, vol. 23, no. 1). Other applications of a more general nature are also indicated. (Received March 10, 1941.)

277. G. E. Albert and J. W. T. Youngs: *The structure of locally connected topological spaces.*

The present paper is a continuation of earlier work (abstract 46-3-138). Cyclic elements are defined and studied in the class L of spaces which satisfy the postulates: the space and the empty set are open; the intersection (sum) of finitely (arbitrarily) many open sets is open; the components of an open set are open. The hyperspace $\mathfrak{X}$ of all cyclic elements of any space $X\in L$ is topologized in such a way that: (1) $\mathfrak{X}$ is a space L, (2) $\mathfrak{X}$ is a strongly continuous image of X, and (3) $\mathfrak{X}$ is acyclic. A subclass H of L is called hereditary if $X\in H$ implies that the hyperspace of X is in H and every true cyclic element of X is a space in H. The class L is a hereditary class which contains the class P of all Peano spaces; however, it deviates widely from P. Hereditary subclasses of L are proposed which approximate more closely the class P. For example: one such class is composed of all spaces L which satisfy the T_0-separation axiom and which are strongly continuous images of the unit line interval. (Received March 10, 1941.)

278. Samuel Eilenberg: *Banach space methods in topology.* I.

For a given topological space X the Banach space $\mathfrak{X}$ of all continuous bounded real functions $\mathfrak{x}$ on X with norm l.u.b. $|\mathfrak{x}(x)|$ is considered. Banach has proved that two compact metric spaces, X_1 and X_2, are homeomorphic if and only if $\mathfrak{X}_1$ and $\mathfrak{X}_2$ are iso-