tions may be regarded as generalizations of the coefficient functions in Taylor series and so his Chapter VII consists of a study of relations between S-functions and special series. Chapter VIII presents a method for the numerical computation of the characters of the symmetric permutation group. The main topic of Chapter IX is stated above but we call attention to Littlewood's interpretation of the topic as a study also of the problem of determining the structure of a group with given characters by an analysis of the characters. The last two chapters connect the theory of group characters for finite groups with the theory of continuous groups, and the book closes with an appendix consisting of tables of the characters of the symmetric groups on $m \leqq 10$ letters and of transitive subgroups.

Littlewood's book is thus primarily an exposition in which the major emphasis is on the theory of group characters for its own sake rather than as a tool for other theories. This is contrary to the attitude of the original investigators in the field, and does result in a book which is notable for the immense number of formulae and formidable computations it contains. The author's justification of his attitude lies perhaps in the remark with which he opens his preface: "Since the discovery of group characters by Frobenius at the end of the last century the development of the theory has been so spectacular and the theory has shown such powerful contacts with other branches of mathematics, both pure and applied, that the inadequacy of its treatment by text-books is rather surprising." The reviewer is inclined to regard as exaggeration the words "spectacular," "powerful" and "surprising," and as unjust the word "inadequacy," but hopes with the author that the special properties of group characters which he derives may have some future interesting applications.

A. A. ALBERT

The Theory of Group Representations. By Francis D. Murnaghan. Baltimore, The Johns Hopkins Press, 1938. 369 pp.

Books on group representations come out in waves, it seems. In short succession, three new books on the subject appeared: Murnaghan's Theory of Group Representations, Weyl's Classical Groups, and D. E. Littlewood's Theory of Group Characters. Perhaps, it is worthwhile to compare the general point of view of these new volumes with the books published during the preceding periods in the history of the theory.

First, group representations were treated as a special chapter of the theory of finite groups, furnishing a powerful method for the study of these groups. The books of Burnside, Blichfeldt, Dickson, Miller, and Speiser belong to this period. Of course, in those days no self-respecting physicist could be seen near the shelves of libraries housing works on group theory. This was the realm of mathematicians, interested in the subject because of its great mathematical importance and its inner beauty.

Suddenly, with the birth of quantum mechanics and modern nuclear physics, everything was changed. There were no self-respecting theoretical physicists anymore who would not know about representations, Schur's lemma, characters, the orthogonality relations, and so forth. The books of this period (Wigner's Gruppentheorie, Weyl's Theory of Groups and Quantum Mechanics, van der Waerden's Gruppentheoretische Methode in der Quantenmechanik) were dedicated half to group theory and half to quantum mechanics, treating the former subject as far as it was needed for the applications to physics. There is no need to describe the immense progress accomplished in this work. Very likely, the books mentioned will be counted among the classics of science.

It is not surprising that the opening of this new field was accompanied by a change of existing values. Certain special groups, the symmetric group and the rotation group suddenly came into prominence. It would not be true to say that they reenacted the story of Cinderella. Such mathematicians as Frobenius, A. Young, I. Schur, Weyl had studied them in detail, but apparently nobody thought of writing books on them, before the applications to physics were found. Of course, the mathematical work was obliterated by the spectacular success of the applications.

What we see today, seems a natural reaction. The mathematicians feel that a theory which admits such applications deserves to be put in the form of books for its own sake. Of course, the needs of physics do not exhaust the mathematical riches. There are more general cases which can be treated, connections to other mathematical theories which can be studied, new questions which can be asked. In this situation, it is hardly an accident that several mathematicians felt the need for a new book on group representations and proceeded to write it, independently of each other. The mathematical theories were treated in all detail, the physical applications were not given. Of course, the three books of Murnaghan, Weyl, and Littlewood differ widely, owing to the different interests of the authors, and their different points of view.

Professor Murnaghan describes the object and scope of his book with the following words: "We have attempted to give a quite elementary and self-contained account of the theory of group representations with special reference to those groups (particularly the symmetric group and the rotation group) which have turned out to be of fundamental significance for quantum mechanics (especially nuclear physics)."

The book begins with a short discussion of the group concept, of linear spaces, and of matrices. No systematical treatment of these subjects is attempted, while, in the later text, the knowledge of the reader is widened, whenever the need arises. We have a representation of a group G by linear transformations of a fixed "carrier space" S, if with every element A of G a linear transformation M_A of S is associated such that we have $M_A M_B = M_{AB}$ for any two group elements A, B of G. Chapter II deals with the question of reducibility. If all the linear transformations of a representation F leave a subspace S_1 of the carrier space S invariant, then the representation is reducible. It breaks up into two representations F_1 and F_2, where F_1 consists of the linear transformations induced by F in S_1, and F_2 of those induced by F in the projection space of S modulo S_1. We have complete reducibility, when S is the direct sum of two invariant subspaces S_1 and S_2; in this case, F is completely determined by the two lower dimensional representations F_1 and F_2. (It may be remarked that the terminology of the book is not the customary one; what Murnaghan terms "completely reducible" is usually denoted as decomposable while the words "completely reducible" are used in another connection.) Then the fundamental theorem of Burnside is proved, which gives a necessary and sufficient condition for reducibility of representations; after that follows the generalization of this theorem by Frobenius and Schur. In order to handle the question of complete reducibility in important cases, Auerbach's theory of bounded representations is developed. The third chapter is concerned with the representations of finite groups and their characters. Each representation is characterized by its character, i.e., the trace of the linear transformation M_A which represents the general element A of G. The main properties of these group characters are derived. This is about as much as is given of the general theory, the greater part of the book is dedicated to the study of special groups, and it is on these special groups that the emphasis of the book is placed. Two types of finite groups are treated in detail, the symmetric and the alternating permutation groups. An important method for dealing with finite groups is the forming of the average of a function of a general group element. In order to extend this method to compact continuous groups, it is necessary to replace the summation by an integration over the group manifold. Particular attention has been devoted to this method of group integration for the unitary and the orthogonal groups. The treatment of the representations of the full linear group includes all continuous representations. In the case of the orthogonal group, not only the one-valued representations but also the two-valued spin-representations are thoroughly studied. In all these cases, many interesting properties of the group characters are derived, e.g., the analysis of the direct product of two representations is given. This is one of Professor Murnaghan's own fields of research. Two short chapters on crystallographic groups and on the Lorentz group and semivectors conclude the book.

This description of the contents will show that the "meet" of Murnaghan's book and Weyl's "Classical Groups" is smaller than one might have expected. Actually, on several occasions, friends of the reviewer expressed regret that two so similar books appeared at the same time. It is therefore necessary to emphasize that they are written for quite different classes of readers, with different ultimate aims. The two authors have different mathematical temperaments, their tastes differ. Professor Murnaghan has kept the modern algebraic view more or less in the background, and sometimes is less insistent on general results than, perhaps, the algebraist would like to see. This should not be understood as a criticism of the book. The avowed aim of the author was to make the theory of representations of the most important groups accessible in a relatively elementary way, and to write a self-contained book, without borrowing much from other theories. It is not the connection of the theory with other parts of algebra and mathematics in which Murnaghan is interested. The need to study the representations of certain groups has arisen, for instance in physics, and he wants to find out as much as possible about these representations, in the most direct way.

Professor Murnaghan's book is written with the clarity which could be expected from the author. All the details are worked out with great care. So the book should prove of high value to mathematicians and physicists who want to become acquainted with the material. Very little special knowledge is required of the reader. The volume can be warmly recommended.

There are a number of misprints, but hardly any of them are of a serious nature. Professor Murnaghan authorizes me to say that a list of misprints is available and will be supplied on request.

RICHARD BRAUER