(or inclusion) relations until a later stage of the inquiry. Generalizations of this kind are not considered in any other elementary text.

To sum up, we have in this book by Bennett and Baylis a textbook on logic, designed for presentation to beginners, and intended as an introduction to modern mathematical, as well as to traditional formal logic. This is a difficult expository problem; and one for which a thoroughly satisfactory solution has not yet been found by anyone. That the book should, under these circumstances, be something of a compromise, is perhaps inevitable. The reviewer has criticized it from an ideal point of view, with reference to the goal to be attained—which, by the way, is of some importance for mathematics; these criticisms are to be taken not as pointing out defects in this book but as suggesting ways in which the next approximation to the goal can be improved. The text is one of great merit; most of the criticisms here made would apply to any similar book the reviewer knows of.

HASKELL B. CURRY

The Theory of Group Characters. By D. E. Littlewood. Oxford University Press, 1940. 8+292 pp.

The theory of group representations and group characters has already been treated very recently in the comprehensive expositions of Hermann Weyl's *The Classical Groups*,[1] and F. D. Murnaghan's *The Theory of Group Representations*. We now have a third treatment. All three differ not only in emphasis but also in the spirit[2] of their approach to the subject.

Littlewood's book is intended by him to give "a simple and self-contained exposition of the theory[3] in relation to both finite and continuous groups, and to develop some of its contacts with other branches of pure mathematics such as invariant theory and the theory of symmetric functions." Thus the first fifty-two pages of his text are devoted to an exposition designed to make it self-contained. Chapter I consists mainly of a discussion of the classical canonical form of a matrix under similarity transformations, the properties of unitary, orthogonal and real orthogonal matrices, the reduction of Hermitian matrices under unitary transformations, and the definition of direct product. Chapter II presents the concept of an algebra and its regular representations, the consequent definition of trace, and the further topics necessary for an understanding of the property

[1] Reviewed in this Bulletin, vol. 46 (1940), pp. 592–595.

[2] Cf. Footnote 4.

[3] Of group *characters*, not of group representations.

that a semi-simple algebra over the complex field may be expressed as a direct sum of total matric algebras. The third chapter includes the topics of permutation groups, the relation of the number of partitions of an integer n to the number of classes in the symmetric permutation group on n letters, and the representations of finite groups by permutation groups. Finally the discussion of the semi-simplicity of a group algebra and the basic theory of group representations are presented in the beginning of Chapter IV.

The somewhat sketchy character of this first part of Littlewood's book would seem to have been dictated by its brevity and to result necessarily in proofs which are neither as complete[4] nor as conclusive as is desirable. Nonetheless, simpler and shorter modern proofs (of the results on matrices and linear algebras which are developed) do exist and could have been employed. They were hardly to be expected, however, in an exposition which uses *spur* for *trace* and *trace* for a special *spur*, which employs *Frobenius algebra* where nearly all others use *group ring* or *group algebra*, which returns to *simple* matric algebra after fourteen years of *total* matric algebra, which presents the theory of group representations so that a major mining operation is necessary to unearth the fundamental *Schur lemma*, and which ignores completely nearly all work of the past fifteen years on matrices and linear algebras.

The remaining portion of the text comprises nearly five-sixths of it, and is devoted to the properties of group characters themselves. They are defined in Chapter IV and a beginning is made there also of the subject matter of Chapter IX, which is a study of the relations between the characters of a group and those of its subgroups. In Chapter V the formula of Frobenius for the characters of the symmetric group is derived and the properties of primitive characteristic units and the Young tableaux are treated. Chapter VI contains a discussion of immanants and S-functions.[5] The author remarks that the S-func-

[4] For example, the demonstration given of the primitive result that a matrix satisfies its characteristic equation uses the similarity theory to prove the result for matrices with distinct characteristic roots and is completed by the statement "if A is any matrix whatsoever we can find a matrix Z such that $A+\mu Z$ has all its characteristic roots distinct and satisfies its characteristic equation. We now take the limit as μ tends to zero, whence A satisfies its characteristic equation."

[5] Richard Brauer has noted the fact, in his review of this book in the Mathematical Reviews, vol. 2 (1941), p. 3, that these functions were obtained by I. Schur in his Berlin thesis of 1941 as the characters of the integral rational representations of the full linear group. See also Brauer's review for certain questions he raises about the validity of Littlewood's discussion of the characters of a group relative to those of its subgroups.

tions may be regarded as generalizations of the coefficient functions in Taylor series and so his Chapter VII consists of a study of relations between S-functions and special series. Chapter VIII presents a method for the numerical computation of the characters of the symmetric permutation group. The main topic of Chapter IX is stated above but we call attention to Littlewood's interpretation of the topic as a study also of the problem of determining the structure of a group with given characters by an analysis of the characters. The last two chapters connect the theory of group characters for finite groups with the theory of continuous groups, and the book closes with an appendix consisting of tables of the characters of the symmetric groups on $m \leqq 10$ letters and of transitive subgroups.

Littlewood's book is thus primarily an exposition in which the major emphasis is on the theory of group characters for its own sake rather than as a tool for other theories. This is contrary to the attitude of the original investigators in the field, and does result in a book which is notable for the immense number of formulae and formidable computations it contains. The author's justification of his attitude lies perhaps in the remark with which he opens his preface: "Since the discovery of group characters by Frobenius at the end of the last century the development of the theory has been so spectacular and the theory has shown such powerful contacts with other branches of mathematics, both pure and applied, that the inadequacy of its treatment by text-books is rather surprising." The reviewer is inclined to regard as exaggeration the words "spectacular," "powerful" and "surprising," and as unjust the word "inadequacy," but hopes with the author that the special properties of group characters which he derives may have some future interesting applications.

A. A. ALBERT

The Theory of Group Representations. By Francis D. Murnaghan. Baltimore, The Johns Hopkins Press, 1938. 369 pp.

Books on group representations come out in waves, it seems. In short succession, three new books on the subject appeared: Murnaghan's Theory of Group Representations, Weyl's Classical Groups, and D. E. Littlewood's Theory of Group Characters. Perhaps, it is worthwhile to compare the general point of view of these new volumes with the books published during the preceding periods in the history of the theory.

First, group representations were treated as a special chapter of the theory of finite groups, furnishing a powerful method for the study