BOOK REVIEWS

Bibliography on Orthogonal Polynomials. By J. A. Shohat, Einar Hille, and J. L. Walsh. Washington, D. C., National Academy of Sciences, 1940. 9+203 pp. $3.00.

This bibliography is concerned principally with Hermite, Legendre, Laguerre, Jacobi, Tchebycheff, and related polynomials. These polynomials, which enter into many phases of pure and applied mathematics, happen to be both important and interesting. Since the time of Legendre (1752–1833) new applications have been requiring new advances in the theory of the polynomials, and new advances in the theory have been finding new applications. Hence it is inevitable that the literature of the subject has grown to vast proportions and will continue to grow. One who, by reason of preference or necessity, wishes to become acquainted with the whole theory or with some part of it is embarrassed not only by the extensiveness of the literature but also by the fact that so many different persons have made contributions in so many different books and periodicals. It is fortunate that the National Research Council of the National Academy of Sciences has collaborated with the three authors to bring forth this bibliography.

The book begins with an alphabetical list of complete titles of 332 periodicals. These periodicals are thereafter referred to by number. This necessitates repeated reference to the list of titles, but it eliminates confusion which results from abbreviations.

The second part of the book gives a code index. Some Roman capitals represent special classical orthogonal polynomials (OP); for example H represents Hermite OP. Other Roman capitals specify the domain of orthogonality and the character of the weight-function. Lower case Greek letters refer to topics, properties, applications, etc.; subheadings are lower case Roman letters and then Arabic numbers. For example α refers to general properties of OP; αd to zeros of OP; Hαd to zeros of Hermite OP; and Hαd4 to bounds for the zeros of Hermite OP. The symbol β refers to expansions of functions; βa1, βa2, $\cdots$, βa19 refer to 19 different classes of functions (L, L^2, L^p, continuous, bounded variation, etc.) to be expanded; and βb1, $\cdots$, βb19 refer to properties of expansions. For example βb14.1, $\cdots$, βb14.9 refer to 9 different types of convergence of expansions such as uniform, absolute, almost everywhere, and in mean; and βb15.1, $\cdots$, βb15.6 refer to summability of expansions. The symbol γ refers to general series of OP not necessarily resulting from expansion of functions; δ to mechanical quadratures; ϵ to interpolation; ζ to approximations; η to the moment problem; θ to probability and statistics; and the code continues until ω is reached.

The third and main part of the book (pp. 25–184) lists 1952 articles by about 600 authors. The full title of each article is given exactly as it appears with the article except in the case of non-Latinic alphabets, when a translation is given. Following each title is an abstract in code. For example, the fourth entry under E. Hille is:

A class of reciprocal functions. [36] (2) 27 (1926) 427–64.

$$\begin{gathered}\mathrm{H}{:}\,\alpha\mathrm{b}4-\mathrm{b}7-\mathrm{b}10-\mathrm{b}13-\mathrm{c}-\mathrm{d}1-\mathrm{d}3-\mathrm{d}4-\mathrm{d}7-\mathrm{f}-\mathrm{f}3-\mathrm{g}-\mathrm{i}-\mathrm{j}-\mathrm{l}1\\ \mathrm{m}1.1-\mathrm{m}1.2-\mathrm{n}2-\beta\mathrm{a}1-\mathrm{a}5-\mathrm{b}12-\mathrm{b}14.1-\mathrm{b}14.2-\mathrm{b}14.3-\mathrm{b}14.4\\ -\mathrm{b}14.8-\mathrm{b}14.9-\mathrm{b}15.1,\ \theta\mathrm{a}2,\ \kappa\mathrm{b},\ \mu,\ \pi,\ \rho.\end{gathered}$$

It takes only a short time to find that the [36] refers to the Annals of Mathematics and to decode the abstract to obtain a clear and precise statement of the topics discussed in the paper.

The fourth part of the book lists 89 books and 39 theses containing theory of OP. Some of these are abstracted and some are not.

The fifth and last part of the book is a topical index which lists works on (i) Hermite OP; (ii) Jacobi OP; (iii) Laguerre OP; (iv) Legendre OP; (v) Applications to theory of approximation, probability and statistics, and mathematical physics; (vi) Series in OP; (vii) Interpolation, mechanical quadratures; (viii) Moment problem; (ix) OP in the complex domain; and (x) Tables and graphs. One feels that these lists are long; there are for example more than 500 papers on series in OP of which the three authors of this bibliography are collectively responsible for 29. Checking over 500 abstracts to find, for example, papers on uniform convergence of series in Hermite polynomials sounds like a formidable task; but actually this task is very much simplified by the code form of the abstracts. One simply looks for Hβb14.1 without being required to read hundreds of pages of verbose abstracts.

So far as the reviewer can determine, the bibliography is accurate and is complete in the sense that it contains all titles which it certainly should contain. There are naturally many books and papers with application to OP which are not cited; for example some on general theory of orthogonal functions which apply to polynomials as well as to other orthogonal functions; some on trigonometric and power series; and some on Sturm-Liouville theory. But to start listing all these and other works which bear upon some ramification of the theory of OP would lead to bibliography without end, and one must feel that the authors have shown good judgment in delimiting their bibliography.

The standard of excellence and usefulness of this book by three capable authors is such that the book will be a valuable adjunct to libraries and to the research equipment of those who work with *OP*. It is appropriate that the book should be used as a model for future bibliographies in other subjects.

RALPH P. AGNEW

Theoretical Hydrodynamics. By L. M. Milne-Thomson. London, Macmillan, 1938. 552 pp. $11.25.

A very comprehensive mathematical treatment of the theory of fluid motion is contained in this text, which presents the lectures by the well known author on this subject to the junior members of the Royal Corps of Naval Constructors at Greenwich. The material presented consists mainly of the theory of the perfect fluid motion, for the first time based consistently on vector notation throughout the text, which thus becomes very concise and brief in the details of mathematical deduction. The great advantages of this form of mathematical writing for this field are obvious to anyone who surveys the wealth of material given in this text. In justice to the author not all the saving in space should be attributed to the use of vector notation; however, instead a considerable amount can be ascribed to the experienced and able use of the descriptive text which is brief but complete in all details. However, the physical interpretations given seem to be treated somewhat too briefly for engineers, and experimental and practical applications are outside of the scope of this theoretical treatment.

A brief review of the contents will bring out the structure of the book. Chapter I contains elementary problems of great variety based on Daniel Bernoulli's theorem. The mathematical tools are introduced in Chapter II on vector analysis, which is followed by the discussion of general properties of fluid motion and such phases of two-dimensional flow as can be treated without recourse to the complex variable. Chapter V introduces the latter and opens up the main part of the book, Chapters VI–XIV, dealing comprehensively with two-dimensional motion in all its aspects from the standpoint of the complex variable and conformal mapping. In addition to the chapter headings of streaming motion, aerofoils, sources and sinks, moving cylinders, theorem of Schwarz and Christoffel, the wake, rectilinear vortices, we find also jets and currents and waves treated extensively in this