

introduced by Mersman (this Bulletin, vol. 44 (1938), pp. 667-673). It is shown that if $\phi(u) \in BV[0, 1]$, and $g(x) = \int_0^1 d\phi(u)/(1+xu)$, then the sequence $\{g(n)\}$, ($n=0, 1, 2, \dots$), is a *C-Folge* in the sense of Hausdorff, and the Hausdorff mean $[H, g(n)]$ is regular if and only if $\phi(1) - \phi(0) = 1$. If $\phi(u)$ is real and monotone and $\phi(1) - \phi(0) = 1$, then $H, g(n) \subset (C, 1)$, and is equivalent to convergence if and only if the function $g(x)$ is bounded away from zero in the half-plane $R(x) > -\frac{1}{2}$. Let $\theta(u) = \sum_{p=1}^{\infty} q_p u^p$, $q_p \geq 0$, $\sum q_p < \infty$. Then $a_n = 1 + n \int_0^1 u^n d\theta(u)$, ($n=0, 1, 2, \dots$), defines a Hausdorff mean $[H, a_n]$ equivalent to convergence if and only if $\sum nq_n < \infty$. (Received January 24, 1941.)

145. J. A. Shohat: *On Bernoulli numbers and polynomials.*

Certain relations are established between Bernoulli polynomials $B_n(x)$ and those of Legendre and symmetric polynomials of Jacobi; and a new proof is given of Jacobi's theorem concerning the expression of $B_{2n}(x)$ in powers of $x(1-x)$. A reasoning of Mandl is utilized and made rigorous in order to obtain in a new way the well known remarkable relation between the Bernoulli number B_{2n} and the zeta-function $\zeta(2n)$. Applications are made to $S_n(p) = 1^p + 2^p + \dots + (n-1)^p$. (Received January 8, 1941.)

146. S. E. Warschawski: *On conformal mapping of infinite strips.*

Let S denote the strip $\phi_-(u) < v < \phi_+(u)$, $-\infty < u < +\infty$ ($\phi_+(u)$, $\phi_-(u)$ continuous), in the w -plane, $w = u + iv$. Let $z = Z(w)$ ($\lim_{u \rightarrow +\infty} \Re Z(w) = +\infty$) map S conformally onto the strip $|y| < \pi/2$ of the z -plane, $z = x + iy$. The principal object of this paper is to obtain asymptotic expressions for $Z(w)$ and its derivative $Z'(w)$ as $u \rightarrow +\infty$. For this purpose two inequalities are established, which are similar to those of L. Ahlfors (Acta Societatis Scientiarum Fennicae, new series A, vol. 1 (1930) p. 10 and p. 16), but which, due to some assumptions regarding the smoothness of the boundary of S , yield sharper estimates for large values of $\Re w_1$ and $\Re w_2$. The asymptotic expressions for $Z(w)$ and $Z'(w)$ are then applied, after suitable transformations, to the study of the order of magnitude of the mapping function of a region R onto a circle in a neighborhood of a finite boundary point P of R . These applications include the cases where P is the vertex of a corner, or of a cusp, or is the asymptotic point of two "concurrent" spirals, and contain as special cases the results presented by the writer in two previous abstracts (42-5-219 and 42-11-409). (Received December 9, 1940.)

APPLIED MATHEMATICS

147. E. S. Allen and Harvey Diehl: *The enumeration of glycols.*

Recently the authors extended the method of Henze and Blair for enumerating certain organic compounds; in particular, the alcohols. This work is used as a basis for a recursive method of enumerating isomeric glycols, that is, compounds whose molecules have two oxygen atoms. The basic numbers which result are the number of structurally asymmetric glycols possessing n carbon atoms and having α enantiomorphically distinct forms, and the number of structurally symmetric glycols possessing n carbon atoms and having α distinct forms, of which β are completely symmetric. In all cases symmetry indicates identity of aspect of the molecule when viewed from the two hydroxyl radicals. (Received December 31, 1940.)

148. Stefan Bergman: *Numerical methods for conformal mapping of polygonal domains.*

The problem of the approximate determination of a conformal mapping can be reduced in certain cases to numerical operations which can be carried out with the aid of devices for computation already in existence or by electrical calculating machines combined, perhaps, with punch machines. In the present paper the calculation of the constants (that is, the branch points of the integrand) in the Schwarz-Christoffel formula (which transforms the half-plane into a polygonal domain) is reduced to the determination of the Fourier coefficients of $[R(\phi)]^n$, and to the solution of a system of linear equations. If the domain is a star domain, $R=R(\phi)$ is the equation of its boundary curve in polar coordinates. Further, methods for the calculation of the resulting integrals are discussed. The method is applied to a technical problem, namely to the problem of torsion in a beam with a polygonal section. The analogous method can be applied for the solution of boundary problems and has important applications in certain problems of aeronautics. (Received January 22, 1941.)

149. F. H. Clauser: *Exact solutions of the equations for the flow of a compressible fluid.* Preliminary report.

Several solutions of the equations given by Tschaplygin for the flow of a compressible fluid are discussed and a method presented for easily accomplishing the transformation of the solutions in the hodograph plane back to the physical plane. (Received December 18, 1940.)

GEOMETRY

150. L. M. Blumenthal and G. E. Wahlin: *On the spherical surface of smallest radius enclosing a bounded subset of n -dimensional euclidean space.*

A short elementary proof is given for the theorem: *If M is any bounded subset of n -dimensional euclidean space E_n with positive diameter d , then there is a unique $(n-1)$ -dimensional spherical surface of smallest radius r enclosing M , and $r \leq [n/2(n+1)]^{1/2} \cdot d$.* In a proof abounding with algebraic difficulties, H. W. E. Jung established these results in 1901 for the case of finite point sets and indicated their extension to infinite sets at the end of his long paper (Journal für die reine und angewandte Mathematik, vol. 123 (1901), pp. 241–257). The simplification offered by the present proof is afforded in large measure by a lemma which shows that *if each $n+1$ points of a subset M in E_n may be enclosed by an $(n-1)$ -dimensional spherical surface of radius r then M itself has this property.* The proof exhibits the geometrically simple nature of the theorem. (Received January 24, 1941.)

151. J. J. DeCicco: *Equilong geometry of differential equations of first order.*

With this paper the study of the equilong geometry of a field of lineal elements is begun. This may be considered to be an analogue of a preceding paper by Kasner and the author in which the conformal geometry of a field is developed. As defined by Kasner, a dual-isothermal family consists of ∞^1 curves which are equilongly equivalent to a pencil of circles (all those tangent to two fixed lines, distinct or coincident). Obviously all dual-isothermal families are equilongly equivalent. It is found that any