BOOK REVIEWS

Einführung in die algebraische Geometrie. By B. L. van der Waerden. (Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, vol. 51.) Berlin, Springer, 1939. 247 pp.

During the past 15 years the branch of mathematics known as algebraic geometry has experienced a considerable revival of interest. This has been due largely to the introduction into this field of methods and results of the modern developments in topology and algebra. Professor van der Waerden has been influential in both aspects of this revival, particularly on the algebraic side. In a series of articles, most of which appeared under the general title "Zur algebraischen Geometrie," he has developed a basic technique for the investigation of the algebraic properties of loci defined by polynomial equations. *Einführung in die algebraische Geometrie* is mainly a systematic presentation of some of the principles established in these articles, together with applications to classical problems of algebraic geometry.

The first three chapters of the book provide an introduction to the general theory which follows. There is first defined the important notion of a space (projective or affine) over a field K. The space is defined in the usual way by means of coordinates, these being elements of K. For the benefit of later developments, K is assumed to be algebraically closed and of characteristic zero, but it is otherwise arbitrary. The remainder of the first chapter is taken up with the derivation of the familiar properties of projective spaces; among the topics discussed are duality, projective transformations and their classification, Plücker coordinates for linear subspaces, correlations, null-systems and linear complexes, and some properties of hyperquadrics and of cubic curves in 3-space.

In the second chapter are introduced the basic algebraic concepts which are used throughout the rest of the book. An algebraic function of the variables $u_1, \cdots, u_n$ is defined as an element of an algebraic extension of the field $K(u_1, \cdots, u_n)$. The properties of these functions are then developed, first in general and then in the case when K is the complex field. The familiar fractional power series expansion of a function of one variable in the complex case is shown to have an exact analogue in a formal power series expansion in the general case. These formal power series play an important part in the discussion of plane algebraic curves, which is the topic of Chapter 3. An algebraic curve in the projective plane (over K) is defined in the customary manner, and the multiplicity of a point of intersection of two curves is determined by factoring a certain resultant connected with the curves; Bezout's theorem follows at once. The formal power series are used to introduce the notion of a branch of a curve; once this has been done the classification of singularities, their reduction by quadratic transformations and the derivation of Plücker's formulas follow quickly. A discussion of cubics and the point groups on them is also included.

Chapters 4 to 8 contain the basic theory of algebraic varieties. (We have translated "Mannigfaltigkeit" as "variety" rather than "manifold" for the latter word has a different meaning in topology.) An algebraic variety is the set of all points of a projective n-space whose coordinates satisfy a set (finite or infinite) of homogeneous polynomial equations. The most important varieties are the irreducible ones; those which cannot be expressed as the logical sum of two or more varieties. With each irreducible variety is associated a nonnegative integer d called its *dimension*. At this point the author pauses to give a precise meaning to that much abused term "general point." A set of $n+1$ elements $(\xi_0, \cdots, \xi_n)$ of some extension of K is called a general point of a variety in n-space if a necessary and sufficient condition that $f(\xi_0, \cdots, \xi_n)=0$, f being a homogeneous polynomial with coefficients in K, is that $f(a_0, \cdots, a_n)=0$ for every point $(a_0, \cdots, a_n)$ of the variety. It is shown that a variety is irreducible if and only if it has a general point. The extended use of the word "point," implied by the above definition, to include $(n+1)$-tuples of an extension of K, is continued throughout the rest of the book. One such point η is said to be a *specialization* of another ξ if every homogeneous polynomial equation satisfied by ξ is also satisfied by η. The notions of general point and specialization are fundamental in the discussion of varieties. Chapter 4 closes with a constructive method of finding the irreducible components of a variety and a proof that every variety over the complex field is a topological complex.

Algebraic correspondences are considered in the fifth chapter. An algebraic correspondence between two varieties M and N is defined as a variety in the product space of M and N. If this variety is irreducible, it has a dimension q. It is shown that in this case to a general point of M (or N) corresponds an irreducible variety of N (M), of dimension b (d); furthermore, if M (N) is of dimension a (c), then $q=a+b=c+d$. The usefulness of this result is demonstrated by applications to the intersections of varieties with linear subspaces of the containing space, to the proof of the existence of 27 lines on a nonsingular cubic surface, and to the construction and investigation of a coordinate system for the d-dimensional varieties of an n-space. In

Chapter 6 the properties of correspondences are combined with the concept of specialization of points to answer the important question as to how the multiplicities of the solutions of an algebro-geometric problem are to be counted. The results are then applied to the investigation of tangent spaces of a variety and the intersection of a variety with hypersurfaces of its containing space.

Chapter 7 treats the general theory of linear systems of varieties of dimension $d-1$ on a variety of dimension d. The elementary properties of these systems, their relation to rational transformations, the behavior of a simple point of the variety under such a transformation, and the Bertini theorems are discussed, much use being made of the general points of the varieties involved. There is also a short discussion of divisor classes, their addition, and complete systems. In the next chapter these concepts are applied to the special case of linear series of point groups on a curve. The usual treatment is given here, commencing with Noether's theorem and leading up to the theorem of Riemann-Roch. The generalization of Noether's theorem for hypersurfaces is also given, together with some applications to space quartics. The last chapter is devoted to an account of Enriques' analysis of neighboring points on a branch of a plane curve, and an examination of their behavior under quadratic transformations.

The most striking difference between the classical introduction to algebraic geometry and the one outlined above is the latter's essential avoidance of all notions of continuity. It is this adherence to purely algebraic processes that enables one to replace the complex numbers of the classical theory by the elements of the arbitrary field K. In carrying over the classical proofs to the more general case it is necessary to have substitutes for the limiting processes which often occurred in these proofs; these substitutes are found in the use of formal power series and the specialization of general points. By applying these processes the known results can be formulated and proved in a way that shows their essentially algebraic nature and also avoids the loose reasoning that has characterized certain branches of algebraic geometry in the past.

In presenting this material van der Waerden has attempted to eliminate all superfluous algebra, in particular the use of ideal theory. The only non-elementary knowledge required of the reader is an acquaintance with the properties of algebraic and transcendental adjunctions of fields, the general elimination theory, and those properties of polynomials centering around Hilbert's basis theorem. By thus avoiding the more intricate algebraic ideas the author is able to keep the geometric background of his arguments in sight at all times, thus avoiding a tendency to over-abstraction which is apt to show itself in this type of work. The way the material is arranged also tends to emphasize the geometry—as each new principle is developed its use is illustrated by one or more applications to geometric problems, and in most cases some additional exercises are provided for the reader.

In his preface the author states that the book was written to provide the reader with the background necessary for the study of the deeper parts of algebraic geometry, especially the theory of surfaces. This requirement is perhaps a little vague, as the word "deep" may mean different things to different people, but by almost any criterion it may be said to have been well carried out. As noted above, the book contains a minimum of algebraic complications, and it certainly gives a good account of the basic notions of the subject. There are a few additional topics which the reviewer would have liked to see discussed, particularly the algebraic function field associated with an irreducible variety and its invariance under birational transformations. This is closely related to the notion of a general point and could easily have been introduced in Chapter 4. However, this is merely a detail; on the whole, we believe that the author has made a very good selection of the material at hand.

Technically the book lives up to the high standard we have been led to expect of its author and publisher. A few misprints and incorrect statements were noticed, but these are all of a trivial nature and can easily be detected and corrected by any conscientious reader. Probably the most serious defect is the lack of an index; this is particularly to be deplored in a book which is to serve as an introduction to a relatively unfamiliar branch of mathematics.

We recommend this book to the attention of every mathematician who is interested in either algebra or geometry, and particularly to those who believe that algebraic geometry is still a backward and unrigorous branch of mathematics. They will find here a clear, systematic exposition of an important new mathematical development, one which will undoubtedly have great influence in enlarging the interest in this fascinating field.

R. J. WALKER

The Decline of Mechanism in Modern Physics. By A. d'Abro. New York, Van Nostrand, 1939. 10+982 pp.

The author of this book is already known as a successful popular writer on science (cf. this Bulletin, vol. 34 (1928), p. 789, for review by T. C. Benton of *The Evolution of Scientific Thought from Newton to Einstein*, Boni and Liveright, 1927).