La Mécanique Ondulatoire des Systèmes de Corpuscles. By Louis de Broglie. Paris, Gauthier-Villars, 1939. 6+223 pp.

Following the historical development of wave mechanics, the opening pages are devoted to an exposition of the analogies between the principles of classical mechanics and those of classical optics. After the general principles of quantum mechanics are formulated, the theory of first integrals is given a particularly extensive treatment. A short sketch of perturbation theory is given, followed by a discussion of the symmetric and anti-symmetric wave functions. "Spin" is then introduced and treated from a non-relativistic point of view. Pauli's exclusion principle is formulated and its experimental justification is discussed. In the last chapter applications are made to the theories of valences, of band spectra, and of the para- and ortho-spectra of hydrogen molecules.

There are a few trivial inaccuracies. The reviewer was greatly perplexed on page 133, until it occurred to him that c and d on line 12 were probably misprints for c^2 and d^2.

Also the reviewer personally does not believe in the cogency of the arguments based on the analogies between optics and classical mechanics. In fact, even as the author himself points out (p. 32), these analogies can not be followed too closely in arriving at the Schroedinger wave equation, which is of the first order with respect to the time rather than of the second order.

These adverse criticisms are, however, of a very minor nature. The purpose of the book, as stated in the author's preface, is to give the reader a bird's eye view of the vast structure of modern non-relativistic wave mechanics. There is no pretension whatever of any new mathematical results or methods; and even for the physicist the work must probably be regarded as purely expository. As such it appears to the reviewer to be outstandingly successful.

D. C. LEWIS

Les Nouvelles Méthodes du Calcul des Probabilités. By L. Bachelier. Paris, Gauthier-Villars, 1939. 8+71 pp.

The reviewer finds it rather difficult to justify the title. In a book dealing with *new* methods in probability and published in 1939 the reader expects to find something about the recent developments (to mention but the new treatment of the law of large numbers), even if the book contains 69 pages only. As it is, it is devoted to results due to the author himself, many dating as far back as 1906. These results are merely stated, which is hard on the reader, unless he decides to go through the original papers cited. (The author acknowledges this difficulty at the close of the book.)

The main features may be summarized as follows. (a) Reducing problems in probability to game-problems, which insures unity of treatment. (b) Introducing the time-element, so that if the number of trials (games) is very large, it may be treated as a continuous parameter, and each game is an element of a continuum.

The book is divided into eight chapters. Chapter I gives the classical formulae of Moivre, Poisson, Laplace. Chapter II is fundamental. It gives, first, a classification of probabilities. The conditions of the game may remain identical at each party (*uniformity*), or they may vary from party to party according to a certain given law; the latter, in turn, may depend for each party on its rank alone (*independence*), or on the eventual results of previous parties (*connectedness*). We further speak of probabilities of *first*, *second*, *third kind* according as the variables under consideration take all values from $-\infty$ to $+\infty$, or one of these variables is limited from above or from below, or from above and from below. The author next gives the probability that the loss of a player who plays t parties be x (assuming independence, but non-uniformity). The "hyperasymptotic formula" for this probability of the first kind is

$$\exp\left\{-\left[\int_0^t \psi'(t)dt + x\right]^2 \Big/ \int_0^t \phi'(t)dt\right\}\cdot\left[\pi\int_0^t \phi'(t)dt\right]^{-1/2}.$$

This replaces Laplace's formula. Here $\psi'(t)$, $\phi'(t)$ (>0) are arbitrary functions determined in each specific problem by its data. Thus, the "*elementary probability of first kind*" is

$$\exp\left\{-[\psi(t)+x]^2/\phi(t)\right\}dx[\pi\phi(t)]^{-1/2},$$

and the total mathematical expectation is

$$E = -\int_{-\infty}^{\infty}\frac{x\exp\left\{-[\psi(t)+x]^2/\phi(t)\right\}dx}{[\pi\phi(t)]^{1/2}} = \psi(t).$$

$\phi(t)$ is called "instability function," so that $\phi'(t)dt$, $\psi'(t)dt$ are respectively the "elementary instability function" and "elementary mathematical expectation" (relative to the time-element dt). In the "symmetric" case $\psi(t)=0$, and the formulas involve $\phi(t)$ only. In case of uniformity $\psi(t)=t\psi_1$, $\phi(t)=t\phi_1$, and the constants ψ_1, ϕ_1 are furnished by the data of the problem. We also find here expressions for the average, quadratic and probable deviations.

The following chapters extend the preceding notions and formulas to more complicated probabilities. Chapter III treats the loss in a game with connected probabilities. Use is made of the Maclaurin expansion for $\psi(x)$ (the existence of such expansion is stated, without further consideration). Chapters IV and V treat probabilities of second and third kinds respectively, under various conditions (unlimited number of plays, non-uniformity, periodicity). Chapter VI deals with several variables, that is, with several players and their respective losses $x_1, x_2, \cdots, x_{n-1}$. Here we find "ellipsoids of probability," on which "lie" $x_1, x_2, \cdots, x_{n-1}$ which at time t have the same probability. The last two chapters deal with mechanics. Chapter VII deals with "cinematic probabilities." A particle is moving (along a straight line or in plane or in space) at random—the motion being characterized by a given instability function; we ask what is the probability of its occupying a preassigned position at time t? The same problem is studied if, in addition to the above random displacement, the particle is activated by a force of attraction toward a fixed center (proportional to the distance). The last chapter (Chapter VIII) treats "dynamical probabilities," that is, the motion of a particle or of a system of particles (probability of preassigned position or of preassigned velocity at a given t) when the acting forces are random forces, totally or partially, for example, regarding their directions.

J. SHOHAT