The nature of the constant A_4 here remains undetermined just as in the papers of Rutledge and Douglass. Whether or not it can be rationally expressed in terms of the constants s_1, σ_1, s_3 and π is an open question. Some light may be thrown on the problem by a further study of the function $\xi_1(x)$ treated briefly by Nielsen.* His definition is as follows,

$$\xi_1(x) = \int_0^1 \frac{\log (1 + t)}{1 + t} t^{x-1}dt, \qquad R(x) > 0. \tag{36}$$

From this equation and (27) it follows that

$$A_4 = 5s_4/16 - \xi_1^{(2)}(1). \tag{37}$$

This in itself, of course, sheds no light but if a relation analogous to (16) could be found involving the function $\xi_1(x)$, it would seem that the question could be answered.

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THE COMPUTATION OF THE SMALLER COEFFICIENTS OF $J(\tau)$

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The purpose of this note is to call attention to the fact that the first twenty-five coefficients $a_0, a_1, \cdots, a_{24}$ in the expansion

$$1728J(\tau) = e^{-2\pi i\tau} + \sum_{n=0}^{\infty} a_n e^{2\pi i n\tau} \tag{1}$$

can be computed with relative ease, making use of H. Gupta's tables‡ of the partition function which extend to $n=600$.

From the multiplicator equation§ of fifth order of $J(\tau)$ we have

$$\begin{aligned} 1728J(\tau) = {} & y^{-1} + 6\cdot 5^3 + 63\cdot 5^5 y + 52\cdot 5^8 y^2 + 63\cdot 5^{10} y^3 \\ & + 6\cdot 5^{13} y^4 + 5^{15} y^5, \end{aligned} \tag{2}$$

with

* N. Nielsen, loc. cit., p. 233.

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‡ *A table of partitions*, Proceedings of the London Mathematical Society, vol. 39 (1935), pp. 142–149; *A table of partitions* II, Proceedings of the London Mathematical Society, vol. 42 (1937), pp. 546–549.

§ Klein-Fricke, *Vorlesungen über die Theorie der elliptischen Modulfunktionen*, vol. 2, p. 61, formula (11), with the values given in vol. 2, p. 64, (5) and vol. 1, p. 154, (1).

$$y = 5^{-3}\frac{\Delta(\tau, 1/5)^{1/4}}{\Delta(\tau, 1)^{1/4}} = e^{2\pi i\tau}\phi(e^{10\pi i\tau})^{6}\phi(e^{2\pi i\tau})^{-6}, \qquad \phi(x) = \prod_{m=1}^{\infty}(1 - x^{m}).$$

On the other hand we have*

$$\begin{aligned}(3)\quad &\sum_{n=0}^{\infty} p(25n+24)e^{2\pi i n\tau}\\ &= \phi(e^{2\pi i\tau})^{-1}e^{-2\pi i\tau}\left\{63\cdot 5^{2}y + 52\cdot 5^{5}y^{2} + 63\cdot 5^{7}y^{3} + 6\cdot 5^{10}y^{4} + 5^{12}y^{5}\right\}.\end{aligned}$$

Combining (2) and (3) we find

$$1728J(\tau) = y^{-1} + 6\cdot 5^{3} + 5^{3}\phi(e^{2\pi i\tau})e^{2\pi i\tau}\sum_{n=0}^{\infty} p(25n+24)e^{2\pi i n\tau}$$

and hence

$$\begin{aligned}(4)\quad & x^{-1} + \sum_{n=0}^{\infty} a_n x^n\\ &= x^{-1}\phi(x^{5})^{-6}\phi(x)^{6} + 6\cdot 5^{3} + 5^{3}x\phi(x)\sum_{n=0}^{\infty} p(25n+24)x^{n}.\end{aligned}$$

Equation (4) may be used to compute the a_n. The first term of the right member can be expanded with the aid of Jacobi's formula for $\phi(x)^3$ while the expansion of $\phi(x)$ in the second term is given by Euler's formula. Thus we may write (4) as

$$\begin{aligned}(5)\quad x^{-1} + \sum_{n=0}^{\infty} a_n x^n = \sum_{n=-1}^{\infty} b_n x^n &+ 5^{3}(x - x^{2} - x^{3} + x^{6} + x^{8} - x^{13}\\ &- x^{16} + x^{23} + x^{27} - \cdots)\cdot\sum_{n=0}^{\infty} p(25n+24)x^{n},\end{aligned}$$

with the values

$b_{-1}=1,$	$b_0=744,$	$b_1=9,$	$b_2=10,$
$b_3=-30,$	$b_4=6,$	$b_5=-25,$	$b_6=96,$
$b_7=60,$	$b_8=-250,$	$b_9=45,$	$b_{10}=-150,$
$b_{11}=544,$	$b_{12}=360,$	$b_{13}=-1230,$	$b_{14}=184,$
$b_{15}=-675,$	$b_{16}=2310,$	$b_{17}=1410,$	$b_{18}=-4830,$
$b_{19}=750,$	$b_{20}=-2450,$	$b_{21}=8196,$	$b_{22}=4920,$
$b_{23}=-16180,$	$b_{24}=2376, \cdots.$		

* G. N. Watson, *Ramanujans Vermutung über Zerfällungsanzahlen*, Journal für die reine und angwandte Mathematik, vol. 179 (1938), pp. 97–128; also H. S. Zuckerman, *Identities analogous to Ramanujan's identities involving the partition function*, Duke Mathematical Journal, vol. 5 (1939), pp. 88–110.

By using (5) and the table of partitions, the values of the a_n may be found by mere additions, subtractions, and a single multiplication by 5^3.

The following list contains the values of the a_n computed with the aid of Gupta's tables. The values for $n \leqq 7$ agree with those given by W. E. H. Berwick.*

$a_0 = 744,$
$a_1 = 1\ 96884,$
$a_2 = 214\ 93760,$
$a_3 = 8642\ 99970,$
$a_4 = 2\ 02458\ 56256,$
$a_5 = 33\ 32026\ 40600,$
$a_6 = 425\ 20233\ 00096,$
$a_7 = 4465\ 69940\ 71935,$
$a_8 = 40149\ 08866\ 56000,$
$a_9 = 3\ 17644\ 02297\ 84420,$
$a_{10} = 22\ 56739\ 33095\ 93600,$
$a_{11} = 146\ 21191\ 14995\ 19294,$
$a_{12} = 874\ 31371\ 96857\ 75360,$
$a_{13} = 4872\ 01011\ 17981\ 42520,$
$a_{14} = 25497\ 82738\ 94105\ 25184,$
$a_{15} = 1\ 26142\ 91646\ 57818\ 43075,$
$a_{16} = 5\ 93121\ 77242\ 14450\ 58560,$
$a_{17} = 26\ 62842\ 41315\ 07752\ 45160,$
$a_{18} = 114\ 59912\ 78844\ 47865\ 13920,$
$a_{19} = 474\ 38786\ 80123\ 41688\ 13250,$
$a_{20} = 1894\ 49976\ 24889\ 33900\ 28800,$
$a_{21} = 7318\ 11377\ 31813\ 75192\ 45696,$
$a_{22} = 27406\ 30712\ 51362\ 46549\ 29920,$
$a_{23} = 99710\ 41659\ 93718\ 26935\ 33820,$
$a_{24} = 3\ 53074\ 53186\ 56142\ 70998\ 77376.$

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* *An invariant modular equation of the fifth order*, Quarterly Journal of Mathematics, vol. 47 (1916), pp. 94–103.