Die Pellsche Gleichung. By Werner Weber. (Deutsche Mathematik, supplement no. 1.) Edited by Theodor Vahlen. Leipzig, Hirzel, 1939. 8+151 pp.

In his preface the author calls this an exhaustive text devoted to the mathematical properties of the Pell equation $t^2-Du^2=h$, $h=\pm 1$, ± 4, that can serve on the one hand as an introduction, but which is intended primarily as a reference work "in dem man endlich einmal alles findet, was anderswo mit schöner Selbstverständlichkeit als bekannt vorausgesetzt wird." He seeks, by deriving the properties of the equation from those of the units in the orders of a quadratic number-field, as it were to clear a path through the forest that has arisen in the literature because of the variety of hypotheses that different writers have imposed on D or on h. But his exposition offers little better than a list of the trees.

The first of four parts is entitled "Pell Equation and Units." It is difficult to see for what class of reader this part is written. A glance at its 146 theorems might lead one to think that it was addressed to utter novices, for all but a few of the proofs are trivial deductions from preceding material. But the triviality is due partly to presupposing at the outset familiarity with the elements of the theory of quadratic fields, with the Dirichlet theory of units, with orders, and at one place with rudiments of ideal theory, topics hardly within the ken of an utter novice, and partly to a too rigid adherence to the Satz-Beweis form of presentation. When we reach the third part of the work we find a helpful résumé, which however would have been much more useful had its essence appeared earlier as an objective and motivation for the first part.

The second part, "Pell Equation and Binary Quadratic Forms," is much better reading, though at times repetitious. Here we find an accurate, fully detailed account of the continued fraction expansion of the roots of reduced indefinite binary quadratic forms, Kronecker forms being treated first and later specialized successively to Gauss forms, to forms with last coefficient -1, and to Dx^2-y^2. The connection of the Pell equation with automorphs of the forms is clearly brought out. Values of h other than ± 1, ± 4 with $|h|<2D^{1/2}$ are treated as they arise naturally in connection with continued fractions.

In the fourth part, under the title "Practical Aids," is a list of published tables connected with the Pell equation and a study of their uses. The arrangement is again unfortunately such as seriously to impair the usefulness of the material. 58 tables are listed under 11 categories on the basis of content. Two of the categories contain precisely the same 8 tables, and a third category merely relists 3 of these 8. Several pages are then devoted to an exhaustive presentation of 78 relations (Verkappungen) between the categories. For example it is carefully noted that tables giving the fundamental solution of $t^2-Du^2=+1$ can be used to find the fundamental solutions of $t^2-Du^2=\pm 1$ and conversely. The portion of this material that is of value could easily be put in much briefer and more accessible form.

On the second page of the preface we are told that elementary algorithms for finding the solutions are not included, neither are "more advanced and special" facts, solutions derived from cyclotomy for example. For history, reference is made to four standard sources, the last of which bears the date 1920. The list of tables includes recent publications, but no other mention is made of the (not very extensive) literature since 1920.

A succinct account of some of the material in this work would fill a need. But the expository form here adopted, with its over-emphasis on trivialities, vitiates the author's care in detail, his accuracy of statement, and the volume's typographical excellence.

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