

An extensive bibliography is given at the end of the booklet.

A few typographical errors were noted. The footnote on page 19 is clearly intended to refer to the first paragraph after the italicized statement of the theorem concerning continuity. In the second formula on page 31, for γ read λ . In the first theorem of section 28, page 46, for (*d'ordre a*, $0 < \alpha \leq 2$) read (*d'ordre α* , $0 < \alpha \leq 2$).

The author assumes that the reader has some knowledge of modern potential theory, especially the theory of capacity. The reviewer feels that even though the pamphlet may not be intended for novices, the discussion of Kellogg's lemma (pages 20-23) could be made clearer by a few prefatory remarks recalling the distinction between Wiener's and de la Vallée Poussin's definitions of capacity.

Despite some imperfections of detail, the pamphlet is unquestionably a notable contribution to the literature on modern potential theory, and will undoubtedly prove highly valuable to many students of this subject.

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Les Fonctions de Matrices. I. Les Fonctions Univalentes. By Hans Schwerdtfeger. (Actualités Scientifiques et Industrielles, no. 649.) Paris, Hermann, 1938. 58 pp.

This monograph should be of particular interest to the student of analysis; the introductory algebraic treatment, in fact, is suggested and delimited by problems in the field of differential equations.

The opening chapter is an exposition of those properties of matrices that are standard equipment for every geometer and algebraist. It is in Chapters II and III that one finds the author's own contributions.

Chapter II deals mainly with the following problem: Given a set of single-valued functions $f(\xi)$, $g(\xi)$, \dots of the real or complex variable ξ , and an n -rowed square matrix A with fixed coordinates in the complex field, it is required to define n -rowed square matrices $f(A)$, $g(A)$, \dots so that the following requirements are satisfied:

1. If $f(\xi)$ is a constant α for every ξ in the set S , then $f(A) = \alpha E$ (E the identity matrix) for every A whose characteristic roots lie in the set S .

2. To the functions $f(\xi) \pm g(\xi)$ and $f(\xi)g(\xi)$ correspond respectively the matrices $f(A) \pm g(A)$ and $f(A)g(A)$.

3. To the function $g(f(\xi))$ corresponds the matrix $g(f(A))$, assuming that the variation of ξ is so restricted that the values η of $f(\xi)$ lie in the domain of definition of $g(\eta)$.

When each function is a polynomial $\sum \alpha_i \xi^i$, the problem is solved by choosing for $f(A)$ the polynomial $\sum \alpha_i A^i$. For a large class of functions the author shows that the definition of $f(A)$ can be obtained from the "polynomial definition" by using an interpolation formula of Hermite. In particular, if $f(\xi)$ is a single-valued function such that $f(\lambda_i)$, $f'(\lambda_i)$, \dots , $f^{(n_i-1)}(\lambda_i)$ exist, where λ_i is a root, of multiplicity n_i , of the characteristic equation of A , then the author obtains for $f(A)$ the formula $f(A) = \sum_{i=1}^m A_i \sum_{r=0}^{n_i-1} (1/r!) f^{(r)}(\lambda_i) (A - \lambda_i E)^r$, where A_i and m are respectively the Frobenius covariants and the number of distinct characteristic roots of the matrix A . In the final section of Chapter II the author shows that from this formula one obtains a definition for $f(A^t)$, t a real or complex variable, if one replaces $f^{(r)}(\lambda_i)$ by $f^{(r)}(\lambda_i t)$. For $f(\xi) = e^\xi$, in particular, the author's formula leads to $e^{A^t} = \sum_{i=1}^m e^{\lambda_i t} A_i \sum_{r=0}^{n_i-1} (t^r/r!) (A - \lambda_i E)^r$.

The function e^{A^t} occupies a central position in Chapter III. Here the author applies the results of Chapter II to the theory of systems of differential equations. As one would expect, the matrix notation enables him to exhibit solutions in a compact and elegant form.

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