estimate of the probability concerned, then the probability calculus is certainly inapplicable here. If one speaks of the sun's rise as an event which is uncertain (like everything else) and therefore as an event which has a probability number, just what physical meaning has such a number? A short review, however, is no place to take issue with Borel's subjectivism; even a reader who disagrees with much of the material can not deny its thought provoking character, and such a reader will very probably enthusiastically endorse some interesting remarks by Darmois in an appended note.

J. L. DOOB

Partielle Differentialgleichungen und ihre Anwendungen auf physikalische Fragen. By B. Riemann. (Edited by K. Hattendorff, with introduction by F. Emde.) Braunschweig, Vieweg, 1938. 12+325 pp.

It is a tribute to Georg Friedrich Behrnard Riemann to have his book written in 1882, itself only a slight revision of the first edition of 1869, reprinted unaltered in 1938. The following explanation is given by Fritz Emde:

"In the course of time a very detailed two volume work on which many authors have cooperated has grown out of Riemann's lectures on partial differential equations (for review of the eighth edition see this Bulletin, vol. 37 (1931), p. 333). The original edition looks quite modest in comparison with this revision. In spite of this engineers and physicists have repeatedly asked for this book which has been out of print for some time, and were justified in doing so since it is a book in which Riemann introduces his readers to the fundamental mathematical ideas in an excellent way and teaches them the methods of solution. For the beginner even today there is hardly a more convenient approach to this subject.

"May Riemann's lectures show their old virtues anew. They will be a credit to any collection of books."

The reviewer cannot, however, refrain from warning against some concepts, for example (see page 9) dx is considered an "infinitely small" quantity.

J. F. RANDOLPH

Geometrie der Gewebe. By W. Blaschke and G. Bol. Berlin, Springer, 1938. 8+339 pp.

During the years 1927 to 1938 there appeared, mostly in the Hamburg Abhandlungen, a series of papers under the general title, "Topologische Fragen der Differentialgeometrie." The authors of "Geometrie der Gewebe" have been the most frequent contributors to this series, and in this book they have systematized and amplified the theory that was built up in these papers.

The basic concept in web geometry is the *sheaf of curves*, a topological image of the portion of a pencil of parallel lines contained in a bounded convex region of the plane of the pencil. An *n-web* is a set of n sheaves of curves, the points on each sheaf constituting the same point set G, such that no two curves of different sheaves have more than one common point. The property of being an n-web is evidently preserved under any topological transformation of G, and the theory of webs concerns itself largely with properties of webs which are invariant under such transformations. The central problem is this: Under what conditions can an n-web be mapped topologically into a set of line segments in a plane, and how can these webs of lines be characterized?

These considerations can easily be extended to configurations of higher dimensionality. Thus in three dimensions we may have a sheaf of surfaces which is the topological image of the intersection of a pencil of parallel planes with a convex region of 3-space, or a sheaf of curves obtained similarly from a bundle of parallel lines. Other higher dimensional analogues are evident, but these two seem to be the only ones whose properties have been studied to any great extent.

The first part of the book is devoted mainly to "hexagonal" webs, in which there exist certain configurations formed by the curves of the web (analogous to the configurations of Desargues or Pappus in projective geometry). The principal result is the following:

Any hexagonal n-web can be mapped onto n pencils of lines in a plane if $n \neq 5$. There exist 5-webs which cannot be so mapped, but these can be mapped onto four pencils of lines and the pencil of conics on their centers.

This theorem is proved for $n = 3$ (the first non-trivial case) by topological methods, and then extended to higher values of n by the use of properties of continuous groups. Similar theorems are obtained for the two 3-dimensional webs mentioned above.

In the remaining two parts of the book the discussion is limited to those webs and transformations for which the defining functions have a suitable number of derivatives. Part II treats of the differential invariants of webs; including among other topics the determination of complete sets of invariants, a characterization of hexagonal webs, and the relation between webs and the theory of parallelism in differential geometry. Extensive use is made here of the properties of Lie groups and the differential operators arising from them. In Part III webs are discussed from the point of view of algebraic geometry, the main tools being Abel's theorem and the properties of Abelian integrals. The *rank* of a web is introduced as the number of ways of setting up certain parametrizations of the curves of the web. The principal theorems in this connection are the following:

If an n-web of lines in a plane has positive rank, it consists of tangents to an algebraic curve (possibly reducible) of class n.

Any 3- or 4-web of maximal rank can be mapped on a web of lines in a plane.

Probably the most striking feature of the book is the way in which so many different branches of geometry are applied to the development of the theory of webs. There are numerous applications of theorems in topology, continuous groups, algebraic geometry, and differential geometry. These theorems are not merely referred to, but proofs of them are given, at least for the special cases which are to be used. As a result the book is almost wholly self-contained, in spite of the wide range of the topics that are touched upon. This feature, and the inclusion of numerous exercises which supplement and extend the expounded material, aid in making the book an excellent introduction to an interesting new chapter in geometry.

R. J. WALKER

Analyse Mathématique. Vol. 2. *Équations Différentielles. Développements en Séries. Nombres Complexes. Intégrales Multiples*. By P. Appell. 5th edition completely revised. (Cours de Mathématiques Générales.) Paris, Gauthier-Villars, 1938. 305 pp.

The author devotes this second volume of his set on Mathematical Analysis to the following topics in the order mentioned: Differential Equations (91 pages, 3 chapters); Developments in Series (80 pages, 2 chapters); Complex Numbers (39 pages, 1 chapter); Multiple Integrals (78 pages, 2 chapters). The written text and the two appended notes are supplemented with 79 well-drawn figures and are divided into 177 sections each with a title and number. A table of contents, but no index, is bound with this second volume. No lists of exercises are included. However the various topics are illustrated by examples worked out by the author and these examples