SHORTER NOTICES

Valeur Pratique et Philosophie des Probabilités. By É. Borel. (Traité du Calcul des Probabilités et de ses Applications, vol. 4, no. 3.) Paris, Gauthier-Villars, 1939. 9+182 pp.

The treatise on probability and its applications, published by É. Borel and distinguished collaborators has been brought to a close by the present book, in which Borel discusses the place of the calculus of probability in man's analysis of the world about him. Borel does not attempt a systematic formal presentation: the book is an agreeably written non-technical discussion of various crucial aspects of the subject. In Borel's opinion, all our knowledge is probability knowledge, and his whole discussion has that point of view. Thus he even considers the problem of evaluating the probability that the sun will rise tomorrow, or the probability *a priori* that astrology is correct. It is difficult sometimes to determine whether probability, as Borel uses the term, refers to an individual's feelings, as measured by odds he might be willing to accept on a certain event, or whether probability refers to an event occurring in accordance with given specified conditions. Thus Borel considers a "reasonable person" R who is told that 12 horses are to race, who knows nothing about the horses, but who is nevertheless asked the probability that a certain horse A will win. According to Borel, R would try to avoid making any judgment in such a vaguely defined situation, but if pressed, could only give the answer 1/12. It seems to the reviewer that if R knows nothing about the horses, he also knows nothing about the probabilities concerned, and that he must necessarily be silent on the whole situation. If the answer 1/12 is proper why should R be reluctant to give it? The answer 1/12 is not really an answer to the question. In fact the individual who answers 1/12 has constructed a new problem: given that an event can occur in 12 ways, the conditions of occurrence being such that the theory of probability is applicable, and such that no one manner of occurrence is favored over another, what is the probability that the event will occur in way A? The answer is of course 1/12; in fact the question was framed with that answer in mind. But the question is not the original one, because the relevant features of the frame of reference of the experiment have been made specific in the second version, while no useful hint of the real frame of reference was given in the first version.

At one point Borel makes a more or less formal definition. Probability is defined subjectively for an individual by the conditions of a bet he would accept. An objective probability value, to Borel, is a value which is the same "for a certain number of individuals equally well-informed on the conditions of the chance event." Since these betting odds are judged by the individuals concerned using their past experience and their desire to profit in the long run, their probability values can be checked by comparison with observed frequencies, which may suggest changed estimates, or even changes in the technique of estimating.

Estimating probabilities is thus the expected combination of experiment and rational thinking. There seems little point in arguing over definitions of probability as an empirical concept. The exact way probability is defined is of little importance as against the way the numbers when obtained are checked and used. The reviewer can see no possible use or meaning in a number which is called the probability that the sun will rise tomorrow. If one adopts the view that an event to which the probability calculus is applicable should have certain characteristics, leading to an actual estimate of the probability concerned, then the probability calculus is certainly inapplicable here. If one speaks of the sun's rise as an event which is uncertain (like everything else) and therefore as an event which has a probability number, just what physical meaning has such a number? A short review, however, is no place to take issue with Borel's subjectivism; even a reader who disagrees with much of the material can not deny its thought provoking character, and such a reader will very probably enthusiastically endorse some interesting remarks by Darmois in an appended note.

J. L. DOOB

Partielle Differentialgleichungen und ihre Anwendungen auf physikalische Fragen. By B. Riemann. (Edited by K. Hattendorff, with introduction by F. Emde.) Braunschweig, Vieweg, 1938. 12+325 pp.

It is a tribute to Georg Friedrich Behrnard Riemann to have his book written in 1882, itself only a slight revision of the first edition of 1869, reprinted unaltered in 1938. The following explanation is given by Fritz Emde:

"In the course of time a very detailed two volume work on which many authors have cooperated has grown out of Riemann's lectures on partial differential equations (for review of the eighth edition see this Bulletin, vol. 37 (1931), p. 333). The original edition looks quite modest in comparison with this revision. In spite of this engineers and physicists have repeatedly asked for this book which has been out of print for some time, and were justified in doing so since it is a book in which Riemann introduces his readers to the fundamental mathematical ideas in an excellent way and teaches them the methods of solution. For the beginner even today there is hardly a more convenient approach to this subject.

"May Riemann's lectures show their old virtues anew. They will be a credit to any collection of books."

The reviewer cannot, however, refrain from warning against some concepts, for example (see page 9) dx is considered an "infinitely small" quantity.

J. F. RANDOLPH

Geometrie der Gewebe. By W. Blaschke and G. Bol. Berlin, Springer, 1938. 8+339 pp.

During the years 1927 to 1938 there appeared, mostly in the Hamburg Abhandlungen, a series of papers under the general title, "Topologische Fragen der Differentialgeometrie." The authors of "Geometrie der Gewebe" have been the most frequent contributors to this series, and in this book they have systematized and amplified the theory that was built up in these papers.

The basic concept in web geometry is the *sheaf of curves*, a topological image of the portion of a pencil of parallel lines contained in a bounded convex region of the plane of the pencil. An *n-web* is a set of n sheaves of curves, the points on each sheaf constituting the same point set G, such that no two curves of different sheaves have more than one common point. The property of being an n-web is evidently preserved under any topological transformation of G, and the theory of webs concerns itself largely with properties of webs which are invariant under such transformations. The central problem is this: Under what conditions can an n-web be mapped topologically into a set of line segments in a plane, and how can these webs of lines be characterized?

These considerations can easily be extended to configurations of higher dimensionality. Thus in three dimensions we may have a sheaf of surfaces which is the topological image of the intersection of a pencil of parallel planes with a convex region