methods; (2) to codify the set of theorems found; (3) to connect them with the work of the rigorists; and (4) to extend the theorems by all possible means." It seems to the reviewer that the author has succeeded well in his attempt.

The nature of the book is indicated by the chapter headings: I, Introduction and Definitions (8 pages); II, The Operator $D\equiv d/dx$ (47 pages); III, Applications to Ordinary Linear Differential Equations with Constant Coefficients (14 pages); IV, Algebraic Theorems (determinants, 18 pages); V, Matrices (12 pages); VI, Systems of Ordinary Linear Differential Equations with Constant Coefficients (26 pages); VII, The Operators $d_1\equiv\partial/\partial x$, $d_2\equiv\partial/\partial y$ (30 pages); VIII, Applications to Partial Linear Differential Equations with Constant Coefficients (18 pages); IX, The Operator $d_i\equiv\partial/\partial x_i$ (6 pages); X, The Noncommutative Operator $xD\equiv\theta$ (8 pages); XI, Solutions in Series (41 pages); XII, The Differential Equation in Mathematical Physics (9 pages); XIII, Initial or Terminal Conditions (10 pages).

The last two chapters belong more properly in a text on differential equations and are so excellent that they should form the introduction to every beginning course in that subject. The first explains the nature of a differential equation and its solution, and gives the most illuminating discussion of these matters that the reviewer has seen in any book. The second shows the meaning of the constants of integration and how to determine them in a wide variety of problems.

The book concludes with three appendices and an index. The third appendix gives the history of operational mathematics and a complete bibliography of the subject from 1765 to the present time.

J. B. SCARBOROUGH

An Introduction to the Theory of Numbers. By G. H. Hardy and E. M. Wright. Oxford, Clarendon, 1938. 16+403 pp.

As the authors have taken pains to describe—too modestly—the nature of their work, we quote from their preface.

"This book has developed gradually from lectures delivered in a number of universities during the last ten years, and, like many books which have grown out of lectures, it has no very definite plan.

"It is not in any sense (as an expert can see by reading the table of contents) a systematic treatise on the theory of numbers. It does not even contain a fully reasoned account of any one side of that many-sided theory, but is an introduction, or a series of introductions, to almost all of these sides in turn. . . . There is plenty of variety in our programme, but very little depth; it is impossible, in 400 pages, to treat any of these many topics at all profoundly."

Those who had the pleasure of hearing the senior author's lectures when he was in the United States ten years ago, will have pleasurable anticipations of what to expect; nor will they be disappointed. The book is like no other that was ever written on the theory of numbers, as an introduction or as a treatise; although Edouard Lucas might have written something like it had he been primarily interested in the analytic theory and were he living today. Some of the topics treated have been frequently discussed in the English and German journals of about the past decade. As might be anticipated from the authors' interests, analysis dominates much of the material. The treatment throughout, even of old things, is fresh and individual.

Owing to the widely varied character of the matter, it is impossible to give a brief summary of the scope of the book, and the following sample must suffice to indicate the contents. The theory of quadratic forms is omitted. Chapters 1, 2 treat the series of primes and the fundamental theorems on divisibility for the rational integers.

Enough of the O, o, $\sim$ notation is explained for the statement (page 9) of the prime number theorem. Solved and unsolved problems on primes are discussed, including what the authors call "unreasonable" conjectures. Chapter 3 is on Farey series and Minkowski's theorem for lattice points within convex contours. In somewhat the same order of ideas, Chapter 23 proves Kronecker's theorem (1887) for n linearly independent irrationals by three methods due respectively to Lettenmeyer, Estermann, and H. Bohr, of which the first is semi-geometrical, the second inductive, and the third analytic. The Minkowski theorems are those on linear forms (the non-homogeneous case included) which appear in the theory of units in algebraic number fields. Attention is called (page 398) to an outstanding unverified conjecture of Minkowski concerning a conceivable generalization of his theorems.

Having mentioned algebraic numbers, we note next what the book contains about them. First, historians will read with interest the digressions on pages 42–45, 180–181. (Incidentally, was Gauss the first to prove, with a slight gap in one lemma, the fundamental theorem?) Chapter 4 is on quadratic irrationals, and includes proofs of the irrationality of e, e^2. Algebraic numbers are defined (Chapter 11) in connection with approximation of irrationals by rationals, and Cantor's theorem on the denumerability of the set of all algebraic numbers is proved; Liouville's construction of transcendentals is given; and finally, the transcendence of e, π is shortly proved, substantially as by Landau. More traditional material on algebraic numbers appears in Chapters 14, 15, where units, primes, and the fundamental theorem are first discussed for certain quadratic fields of class number unity, leading up to the simplest classical examples for which the fundamental theorem fails. Euclidean quadratic fields (those in which a G.C.D. process holds) have recently received much attention; it is proved that the number of real quadratic euclidean fields is finite. An uncommon application of algebraic fields is Western's proof (page 223) of a primality test due to Lucas. Ideals are defined for quadratic fields and are illustrated in one usual way by point-lattices, but their theory is not developed.

Under congruences and allied topics (Chapters 5–8), in addition to the customary material of an introduction, some more recent novelties are included, such as the divisibility of $2^{p-1}-1$ by p^2, p prime, factors of Mersenne numbers, Wolstenholme's (Waring's) theorem, an inductive proof of the Clausen-von Staudt theorem, Bauer's identical congruence, and applications of Euler's ϕ-function to Gauss and Ramanujan sums. Without attempting an exposition of the Gaussian theory of cyclotomy, the authors give Richmond's construction for the regular 17-gon, with a slight suggestion making "it plain to the reader (as is plausible from the beginning)" why the synthetic magic works. Obviously a good deal of the material in these chapters, as in most of the others, is intended to soften the shock some readers may be expected to experience if and when they follow up these suggestive introductions by systematic study.

Some of the simplest arithmetical functions are discussed in connection with the classic inversion formulas in Chapter 16. Opportunity is taken to prove Euclid's theorem on perfect numbers. Chapter 17 on generating functions of arithmetical functions gives a brief introduction to the customary topics under this head. The following chapter discusses the order of certain of the simpler arithmetical functions —number and sum of divisors, Euler's function, the number of lattice points in a circle, without excessive refinements of the analysis. This is followed (Chapter 19) by a sketch of the theory of partitions as a typical problem in additive arithmetic, enough of Jacobi's identity being proved to yield some of the more interesting classical theorems. Graphical proofs are also given, and one of Ramanujan's congruences is proved. The chapter concludes with the proofs of two Rogers-Ramanujan identities and a hint of the related continued fractions.

Chapter 20 on representations as sums of 2 or 4 rational integer squares provides an opportunity to sketch the arithmetic of quaternions, and serves as an introduction to Waring's problem for cubes and fourth powers in Chapter 21, where Tarry's problems, sums $\sum \pm x_i^k$, and equal sums of two kth powers are also briefly considered. Any reader of this notice who is looking for something hard to do is invited to observe page 336, (f). The notes on pages 334–337 summarize the main results up to 1938 on Waring's problem.

Chapter 22, the third in the book on the series of primes, follows Landau's proofs of Tchebychef's theorems, and pushes the analysis a little further to obtain Mertens' approximation to $\prod_{p \leqq x} (1-p^{-1})$, p prime. The chapter concludes with a note on "round" numbers and a theorem of the "almost all" type on the order of the number of divisors of n.

As the last of the authors' introductions, we mention Chapter 13 on certain Diophantine equations ($x^n+y^n=z^n$, $n=2, 3, 4$), the expression of m as a sum of rational cubes, $x^3+y^3=3z^3$, and equal sums of two cubes.

The foregoing sample from the two dozen chapters covering 400 pages may give some idea of the extraordinary richness of the material, and suggest the justice of the authors' own characterization of their work as "a series of introductions" to a vast and many-sided theory. They have presented these introductions in a manner that should stimulate a reader to continue beyond some of them; and it seems safe to say a great deal more than what they themselves say, "we can hardly have failed completely, the subject-matter being so attractive that only extravagant incompetence could make it dull." The book is anything but dull; in fact it is as lively as the proverbial (not the English) cricket.

E. T. Bell

Le Concours de l'Analyse Mathématique à l'Analyse Expérimentale des Faits Statistiques. By Georges Hostelet. (Actualités Scientifiques et Industrielles, no. 585; Le Progrès de l'Esprit; Exposés publiés sous la direction de L. Brunschvicg.) Paris, Hermann, 1937. 70 pp.

The first section can be summed up by stating that the experimenter believes that the basic laws on which statistics are founded have been proven by the mathematician, while the mathematician believes that these laws have been demonstrated by the experimenter.

In the next section the author states that the mathematical analysis of statistical facts are contained in the intuitive-empirical notion of species, which implies notions of means, errors of means, frequencies and limits of these errors. He gives the conditions, which a statistical fact must satisfy in order that the arithmetic mean signifies the best value, defines the analytic and synthetic indicator of the degree of correlation, outlines the conditions necessary for linear correlation and considers that mathematical analyses are auxiliaries to experimental analyses of measurable facts when measurements present numerical relations expressible in explicit functions.

Section three contains postulates on which mathematical and experimental reasoning depend, answers to objections Fréchet made concerning probability and its use in the analysis of statistical facts, the necessary steps for verifying an hypothesis and an explanation of the difference between mathematical and experimental reasoning.

Chapter IV presents the three modes of scientific investigation, viz: the empirical-intuitive mode, the deductive-abstract mode and the abstract-experimental mode and points out their limitations.