theorems of the theory of operators, including their reduction by linear manifolds, are found in Chapter 4. The fifth and last chapter deals with the problem of finding the inverses, if any, of a bounded linear operator A—that is, the operators X such that XA or AX is the identity. In both Chapters 4 and 5 special attention is devoted to examining the relations between operators, matrices, and infinite systems of linear equations. According to the author's preface, the spectral theory for *bounded* operators will be reached in the succeeding volume.

The topics included in the book are presented from a purely mathematical point of view in a clear and lively style. The applications to the theory of matrices and equations, which are largely implicit in certain of the more abstract treatments, are elaborated here with a wealth of detail which renders them unusually accessible to the student. The author's approach to the modern theory of operators is obviously a cautious one, presumably because of his desire to keep the reader on ground which shall appear as nearly familiar as possible at every stage. In the absence of any indication of the methods which Professor Julia proposes to use in the treatment of the spectral theory or of unbounded operators, the reviewer cannot judge whether such caution is excessive or not. Nevertheless, there would be obvious and important advantages in a more rapid approach to these central topics, beyond which lie the really difficult parts of operator-theory.

The reviewer has noted very few misprints. On page 72, lines 11–14, $\leqq$ should be replaced by $<$; and on page 115, line 11, $<$ by $\leqq$. The attribution to A. Weil of the theorem stated and proved on pages 23–25 is incorrect; in the cases of $\mathfrak{H}_0$ and $\mathfrak{L}_2$ the theorem has long been known; in the abstract form it is a special case of a theorem of Banach (*Théorie des Opérations Linéaires*, page 80, Theorem 5); and the proof given here is essentially that published by von Neumann in Mathematische Annalen, vol. 102 (1929), page 380, footnote.

M. H. STONE

Nouveaux Éléments d'Analyse. Calcul Infinitésimal. Géométrie. Physique Théorique. Vol. 2: *Variables Complexes.* By A. Buhl. Paris, Gauthier-Villars, 1938. 214 pp.

This book is an ably written volume with a view towards a synthesis of the fields indicated. The main purpose of the work is to give young minds not possessed of an extensive mathematical knowledge a quick access to certain higher fields. Throughout the book pertinent references to significant papers and books are given, thus enabling the reader to study in greater detail any field in which he may become interested.

In Chapter 1 the author points out the importance of the equation $\phi(ax)=b\phi(x)$ (É. Borel) for the consideration of periodic, analytic, and nonanalytic functions. Emphasis is placed on the fact that it is possible to pass from the simplest cyclic functions to periodic functions. Certain quantitative generalizations of cyclic functions are given, together with the indication that generalizations of such type for the simplest Schrödinger equations lead to the study of quantitative configurations of the atom. The author then introduces analytic functions and indicates that trigonometric series may be connected in a profound manner with analytic considerations. The chapter concludes with the study of Abel's functional equation, Julia's iterations of rational fractions, automorphic functions (Henri Poincaré), and the homographic group. Among the problems reference is made to the Gibbs phenomenon.

In Chapter 2 the point of view is mainly that involved in the treatises of Picard and Goursat. An analytic function is defined by the property that (under suitable conditions) its integral along an arc AB depends only on the extremities A, B. The author states that a definition of such type is in harmony with modern science;

that it leads also, for example, to the equations of Monge-Ampère, and to the electromagnetic equations of Maxwell. It is also indicated that monogeneity comprises analyticity and quasi-analyticity. The present reviewer would add that on the basis of his own work (see his recent paper in the Annales de l'École Normale Supérieure, as well as a forthcoming memoir in the Acta Mathematica) it can be asserted that monogeneity comprises a considerable number of other significant notions as well. The author touches upon the Cauchy point of view regarding analytic functions, the Cauchy-Riemann differential relations, and the total differential of Stolz-Fréchet; he further considers Laplace equations and their connection with some problems of mathematical physics. The chapter concludes with a discussion of conformal mapping. Among the problems of this chapter reference is made to the notion of the areal derivative (Pompeiu), together with appropriate representations (involving such derivatives) of functions with the aid of certain integral formulas. In this field important contributions have been made by Pompeiu, Montel, and Evans.

In Chapter 3 the author discusses uniform convergence, analytic continuation, entire functions, Liouville's theorem (together with certain modern ideas), the calculus of residues (with an application to the geometry of conics), meromorphic functions, and nonuniform integrals of the first kind.

In Chapter 4 the apparently overlooked importance of the Mittag-Leffler function $E_\alpha(x)$ is pointed out. This function is studied in some detail. On page 121 an important Mittag-Leffler expansion involving $E_\alpha(x)$ is formulated. The notion of quasi-analyticity is discussed very briefly. Finally, the points of view of Cauchy, Weierstrass, and Riemann are compared, and the author points out that the first point of view is apparently more general than the second (this may be substantiated by Borel's book of 1917, as well as by the contributions of the present reviewer as cited above).

Chapter 5 is dominated by the ideas of Darboux and Appell. Elliptic functions are treated as functional invariants of the homographic group (one variable). The author then comes to the projective treatment of angles and to the geometry of Cayley. Reasons are given for considering electromagnetism as a geometry (Barbarin and Buhl). Cubics are considered under homographic transformations, as well as their uniformization (Poincaré). The author then touches upon "projective universes" (Cartan, 1937) and indicates that certain fundamental fields in physics depend initially on projective considerations.

In Chapter 6 the methods of summation of Cesàro, Mittag-Leffler, and Borel are discussed. The author establishes the fact that the study of summability by Taylor's polynomials leads to the consideration of a double integral generalizing the simple Cauchy integral. On page 173 he gives an important formula leading to methods of summability whenever certain conditions are satisfied.

The final chapter, 7, is dominated by the ideas of Hermite and is largely based on books on quantum mechanics by de Broglie, Weyl, Wigner, and von Neumann. The author discusses the equations of Maxwell, the equations of dynamics, and the Jacobi-Hamilton system. He points out that to the Hermitian origin of the equations of Maxwell corresponds the Hermitian origin of the geometry of Cayley. The chapter is concluded with the indication that the considerations of Schrödinger are connected in a natural way with the field of differential equations, that there is a new connection between the gravitational theory and wave mechanics, and that in a certain sense electricity possesses dual character.

The problems at the end of each chapter are well chosen, and some of them are highly instructive.

W. J. TRJITZINSKY