

bounded variation illustrate a combination of the two ideas. Brief mention is also made of the theorem on polynomial approximation to continuous functions, of Tchebycheff polynomials, and of summability of sequences.

Part II, beginning with integration, applies the theory. Brief treatments of the Riemann, Lebesgue, and Stieltjes integrals are given, and mention is made of the Denjoy totalization process. Chapter 2 takes up trigonometric series. The importance of such series in leading to a general conception of function (attributed to Riemann) is indicated. There follow remarks concerning the determination of the coefficients, uniqueness, tests for convergence, the Fejér method of summation; then a short note on general orthogonal functions. Chapter 3 deals with quasi-analytic functions (dear to the heart of Denjoy). He sets the general problem leading to such functions, and states the solution of Carleman. A last chapter discusses functionals: continuity, linearity, differential of a functional, functions of sets.

Denjoy had looked upon the handiwork of those who have made sound contributions to real variable theory, and saw that it was good.

I. M. SHEFFER

Sur les Fonctions d'une Variable Complexe Représentables par des Séries de Polynômes.

By M. Lavrentieff. (Actualités Scientifiques et Industrielles, no. 441.) Paris, Hermann, 1936. 60 pp.

This brochure is in the section of its series on *The Theory of Functions*, edited by Paul Montel. Baire found the condition that a function of a real variable be of class one (representable as a series of continuous functions). Using the approximation of continuous functions by polynomials (Weierstrass), one may say that Baire determined when a function is the sum of a series of polynomials. The extension of this problem to the complex plane is the subject of Lavrentieff's work. He treats the problem from its classical beginnings to the latest advances, thus providing a valuable résumé.

Chapter 1 is introductory. Baire's work is taken as starting point, after which the first general advance to the complex plane, the well known Hilbert-Runge theorem, is stated and proved. In the short second chapter are stated some needed theorems on the correspondence of boundaries under conformal mapping.

One of the problems dealt with in Chapter 3 concerns the convergence of sequences of (holomorphic, in particular, polynomial) functions that are bounded in their set, together with a uniqueness theorem (to the effect, for example, that two functions which are limits of such sequences are identical in a given region if they coincide on certain point sets of the boundary). Another problem treated is that of finding necessary conditions and sufficient conditions that a function have a polynomial expansion (of given type) on a closed point set. An extension to harmonic sequences is indicated.

In the last chapter special point sets M and M^* (which space does not allow to describe adequately) are introduced, and it is shown that these sets are important in the statement of general theorems on the representation of holomorphic functions by series of polynomials.

I. M. SHEFFER

Diophantische Approximationen. By J. F. Koksma. (Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 4, no. 4.) Berlin, Springer, 1936. 8+157 pp.

It is remarkable material on diophantine approximations with which the author presents us in this volume. The bibliography, which seems to be unusually complete,

contains about eight hundred separate titles. It is needless to emphasize the importance of such a survey for those working in this field.

Let us only mention a few of the main theories considered by the author. The theory of diophantine approximations has been taken in its widest sense, so that one also obtains information about neighboring theorems and applications. The book begins with a short survey of the field to obtain a classification of the main problems. One then finds a discussion of Minkowski's analysis, his Geometry of Numbers and applications to homogeneous and non-homogeneous linear forms. At this point one should also mention the account of Blichfeldt's methods. The properties of continued fractions are then discussed together with the neighboring theories, namely, of irrationality and transcendency and their measures. The approximation theorem of Kronecker is discussed at length. The distribution of real numbers (mod 1) covers two chapters and one finds accounts of the investigations of Weyl, Vinogradoff, van der Corput, and others. The final chapter is devoted to the theory of diophantine inequalities.

OYSTEIN ORE

Neuere Fortschritte in den exacten Wissenschaften. Fünf Wiener Vorträge. Ser. 3. *Vitamine und ihre Bedeutung.* By E. Späth. *Die physikalischen Entdeckungen der letzten Jahre.* By H. Thirring. *Extreme Versuchsbedingungen als Quelle des Fortschrittes.* By H. Mark. *Prinzipielle Fragen in der modernen Physik.* By W. Heisenberg. *Einige neuere Fortschritte in der exacten Behandlung sozialwissenschaftlicher Probleme.* By K. Menger. Leipzig and Vienna, Franz Deuticke, 1936. 132 pp.

Five scholarly lectures by experts in physiology, physics, and social science are here presented. The first, on vitamins, is good but not novel. Professor Thirring's exposition of the discovery of the elementary particles of physics is systematic and up to his usual high standard. The third lecture is especially timely for mathematicians and theoretical scientists since it gives an account of the precision of the experimental determination of the major empirical constants which enter into physical theories. Perhaps the most unique and interesting of the lectures is by Heisenberg. He argues against a future reduction of the statistical aspects of quantum theory to causal laws. The final essay offers suggestive and original evidences of the application of mathematics and mathematical logic to social science. These lectures maintain a high level of excellence and merit the attention of serious students.

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