his derivation of the general form of the characteristic function of an indefinitely divisible distribution law. He then goes on to consider stable distribution laws and related topics. Chapter VIII extends various results (Liapounoff theorem, law of iterated logarithm, and so on) known for sets of mutually independent chance variables, replacing the condition of independence by certain more general conditions. In Chapter, IX Lévy discusses measure properties of developments in continued fractions, using the suggestive terminology of probability.

Lévy's book can be recommended only to advanced students of probability who already have some familiarity with the topics treated. Other readers will merely be exasperated by his confidence that they have his own unsurpassed intuitive grasp of the subject. The student preparing himself to do research in probability, however, will find here the latest results in an important field, derived in a way which stresses methods rather than details.

J. L. DOOB

Les Lois des Grands Nombres du Calcul des Probabilités. By L. Bachelier. Paris, Gauther-Villars, 1937. 7+36 pp.

Bachelier believes that many of the results in his books and papers have been unnoticed by later writers. In this book he restates many of these results (without proofs). He first considers the Bernoulli case: independent trials, each having only two possible results, having probability p and $1-p$. He then generalizes in various directions, letting p vary from trial to trial, and so on. The formulas are of asymptotic character, approximations which improve as the number of trials increases. As an example of their general character, we give one result. Bachelier finds that (in the Bernoulli case) if μ_1 trials are made, and if we consider the difference between the number of times the event with probability p has occurred and its expected value, then the probability that this difference will return to 0 before μ further trials are made is $(2/\pi)$ arc tan $(\mu/\mu_1)^{1/2}$.

J. L. DOOB

Theorie der Orthogonalreihen. By Stefan Kaczmarz and Hugo Steinhaus. (Monografje Matematyczne, vol. VI.) Warsaw, 1935. vi+298 pp.

The present volume of the excellent Polish Series is devoted to the theory of general orthogonal functions of a single real variable. Desiring not to increase the size of the volume without proportionally increasing its usefulness, the authors omitted almost completely the theory and applications of special orthogonal functions including that of orthogonal polynomials, and concentrated their attention on general orthogonal functions as a tool in pure mathematics. Even in this restricted field no claim is made for "encyclopaedic completeness." Despite these somewhat severe restrictions the authors succeeded in presenting a very interesting material widely scattered in the literature, including also some new contributions of their own.

The book consists of eight chapters followed by a bibliography containing 129 items. Chapter 1 (pp. 1–30) gives a brief exposition of general notions of abstract spaces, and linear operations and functionals which serve as a most important tool in the subsequent developments. Chapter 2 (pp. 37–60) introduces the fundamental concepts of orthogonality, completeness, closure, and best approximation. Chapter 3 (pp. 61–102) discusses general orthogonal series in L^2 including theorems of Müntz and of Riesz-Fischer, and Parseval's identity. Chapter 4 (pp. 103–148) treats of various examples, with particular attention given to orthogonal systems of Haar and of Rademacher and to the "probabilistic" interpretation of the latter. Chapter 5 (pp. 149–194) is devoted to the theory of convergence (almost everywhere, unconditional, $\cdots$), divergence, and summability of orthogonal series. The climax of this chapter is reached in an elegant proof of the fundamental theorem of Rademacher-Menchoff. A systematic use of "Lebesgue's functions" associated with orthogonal expansions deserves a special mention. Chapter 6 (pp. 195–242) deals with orthogonal expansions in various spaces (L^P, C, M) different from L^2. Among various topics treated here we mention the relationships between the closure and completeness of an orthogonal system; the theorems of Young-Hausdorff and of Paley; the theory of "multipliers" transforming orthogonal expansions of functions of various classes into each other; and a discussion of various singularities which occur in orthogonal expansions. Chapter 7 (pp. 243–260) reveals various remarkable properties of "lacunary" series. Chapter 8 (pp. 261–298), the last chapter, is of somewhat mixed character, being devoted partly to biorthogonal expansions, and partly to polynomials orthogonal relative to a given weight-function.

The exposition, which is in general clear and concise, in some places shows a tendency to be either somewhat vague, or so condensed that it will be difficult to follow for a reader who is not well versed in the field. The number of misprints (in addition to those mentioned in a list of 16 Errata) and of slips of pen or thought is not entirely negligible. Thus on page 6 the reader is told that every point set can be decomposed into a sum of a perfect set and of an at most denumerable set; on page 19 the norm of the functional $\int_a^b dg$ is stated to be $\int_a^b |dg|$; the condition [874] on page 280, which is sufficient for the completeness of the system of orthogonal polynomials relative to the weight function $w(t)$, is obviously not satisfied in the case of Laguerre polynomials, contrary to the assertion preceding this condition. In order to avoid footnotes the authors are using a new scheme of cross references, which, according to the reviewer's experience, does not represent an improvement over the customary system.

J. D. TAMARKIN

Introduction à la Théorie des Fonctions de Variables Réelles. Parts I and II. By Arnaud Denjoy. (Actualités Scientifiques et Industrielles, nos. 451 (55 pp.) and 452 (57 pp.).) Paris, Hermann, 1937.

These are two of the brochures in the section on *Sets and Functions*, under the editorship of Denjoy, who here writes Parts I and II on the introduction to real function theory. Having himself gained important results (on derived numbers, totalization, and so on), Denjoy has paused to scan the field of sets and real variables. From the brevity of each brochure it is clear that his treatment of the various topics is necessarily skeletal (it omits all proofs), but, we believe, is interesting and successful.

Early in Part I he advances reasons of a physical nature for studying nonanalytic, even discontinuous, functions. Then comes a foremention of names and topics to be considered: Cauchy, Abel, Riemann (on convergence and integration), Cantor (sets, transfinite numbers), Baire (classification of functions), Borel, Lebesgue (measure and integration), $\cdots$; and a mention of general analysis. Chapter 2 deals with the geometry of Cartesian point sets, that is, familiar point set theory (including measure). The author here makes distinction between descriptive (topological) ideas and metric ideas, which distinction is also carried over to Chapter 3 on the analysis of functions. Examples of descriptive notions are continuity, convergence, the Baire classification; of metric notions, derived numbers, differentials. Functions of