

theorem quoted gives also the simplest condition, which is that the functions u and v , where $f = u + iv$, possess a Stolz differential and satisfy the Cauchy-Riemann differential equations. If the Stolz differential is assumed, then it is also sufficient either (a) that the difference quotient $\Delta f/\Delta z$ have the same limit for any two distinct directions, or (b) that $\arg \Delta f/\Delta z$ have the same limit for three distinct directions, or (c) that $|\Delta f/\Delta z|$ have the same limit in three directions, but in the latter case \bar{f} may be monogenic instead of f . The theorems giving sufficient conditions for the holomorphy of a function in a region depend upon sufficient conditions for the expression of the integral $\int f(z) dz$ around a rectangle with sides parallel to the axes in the form of the double integral

$$- \iint \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) dx dy + i \iint \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) dx dy$$

and the fact that if $\overline{|\Delta f/\Delta z|}$ is finite on a measurable set, then the Stolz differential of f exists except on a set of measure zero, various theorems being obtained by giving sufficient conditions for the application of these results and the use of Morera's theorem.

In the main the monograph is a brief presentation of the author's investigations in these questions, which may be justifiable, but involves the possibility of overlooking simplifications in presentation. For instance, the three supplementary sufficient conditions for the monogeneity of a function at a point are geometrically intuitive if use is made of the Kasner circle.* However, the monograph is informative and suggestive; especially might one call attention to the remark in the introduction that, while many theorems have a form which involves only the complex variable situation, it has so far been necessary to use in their proof deep-seated methods of the modern theory of real functions, and that it would be interesting and desirable to derive these same theorems without departing from the setting in which they are stated.

T. H. HILDEBRANDT

Théorie de l'Addition des Variables Aléatoires. By P. Lévy. (Monographies des Probabilités, publiés sous la direction de E. Borel, no. 1.) Paris, Gauthier-Villars, 1937. 17+328 pp.

Professor Lévy's book is the first of a series of monographs on probability and its applications. This first monograph covers the field of Lévy's special interest, to which he has made valuable and often definitive contributions: certain questions connected with asymptotic problems in probability theory. The first four chapters include a general introduction to probability, designed to make the book complete in itself. Chapter V contains theorems related to the Gaussian law—Cramer's recently proved theorem and various extensions of the Liapounoff theorem. In Chapter VI, Lévy discusses series whose terms are mutually independent chance variables. He includes an important theorem on dispersion, recently derived by himself and Doeblin. Chapter VII considers a chance variable x_t (depending on the parameter t) whose increments in non-overlapping t -intervals are independent. The treatment is somewhat confusing in that measurability considerations are omitted and it is not clear whether x_t considered as a function of t has been proved to have (almost certainly), at worst, jumps at its points of discontinuity. Lévy gives a considerably simplified version of

* This Bulletin, vol. 34 (1928), p. 561.

his derivation of the general form of the characteristic function of an indefinitely divisible distribution law. He then goes on to consider stable distribution laws and related topics. Chapter VIII extends various results (Liapounoff theorem, law of iterated logarithm, and so on) known for sets of mutually independent chance variables, replacing the condition of independence by certain more general conditions. In Chapter, IX Lévy discusses measure properties of developments in continued fractions, using the suggestive terminology of probability.

Lévy's book can be recommended only to advanced students of probability who already have some familiarity with the topics treated. Other readers will merely be exasperated by his confidence that they have his own unsurpassed intuitive grasp of the subject. The student preparing himself to do research in probability, however, will find here the latest results in an important field, derived in a way which stresses methods rather than details.

J. L. DOOB

Les Lois des Grands Nombres du Calcul des Probabilités. By L. Bachelier. Paris, Gauthier-Villars, 1937. 7+36 pp.

Bachelier believes that many of the results in his books and papers have been unnoticed by later writers. In this book he restates many of these results (without proofs). He first considers the Bernoulli case: independent trials, each having only two possible results, having probability p and $1-p$. He then generalizes in various directions, letting p vary from trial to trial, and so on. The formulas are of asymptotic character, approximations which improve as the number of trials increases. As an example of their general character, we give one result. Bachelier finds that (in the Bernoulli case) if μ_1 trials are made, and if we consider the difference between the number of times the event with probability p has occurred and its expected value, then the probability that this difference will return to 0 before μ further trials are made is $(2/\pi) \arctan (\mu/\mu_1)^{1/2}$.

J. L. DOOB

Theorie der Orthogonalreihen. By Stefan Kaczmarz and Hugo Steinhaus. (Monografie Matematyczne, vol. VI.) Warsaw, 1935. vi+298 pp.

The present volume of the excellent Polish Series is devoted to the theory of general orthogonal functions of a single real variable. Desiring not to increase the size of the volume without proportionally increasing its usefulness, the authors omitted almost completely the theory and applications of special orthogonal functions including that of orthogonal polynomials, and concentrated their attention on general orthogonal functions as a tool in pure mathematics. Even in this restricted field no claim is made for "encyclopaedic completeness." Despite these somewhat severe restrictions the authors succeeded in presenting a very interesting material widely scattered in the literature, including also some new contributions of their own.

The book consists of eight chapters followed by a bibliography containing 129 items. Chapter 1 (pp. 1-30) gives a brief exposition of general notions of abstract spaces, and linear operations and functionals which serve as a most important tool in the subsequent developments. Chapter 2 (pp. 37-60) introduces the fundamental concepts of orthogonality, completeness, closure, and best approximation. Chapter 3 (pp. 61-102) discusses general orthogonal series in L^2 including theorems of Müntz and of Riesz-Fischer, and Parseval's identity. Chapter 4 (pp. 103-148) treats of various examples, with particular attention given to orthogonal systems of Haar and