SHORTER NOTICES

Leçons sur les Séries Hypergéométriques et sur quelques Fonctions qui s'y Rattachent. I. *Propriétés Générales de l'Équation d'Euler et de Gauss.* By É. Goursat. (Actualités Scientifiques et Industrielles, no. 333.) Paris, Hermann, 1936. 92 pp.

Within a comparatively limited space and assuming only the fundamental conceptions about linear differential equations in the complex domain, the author develops the principal facts about the hypergeometric equation

$$x(1 - x)y'' + [\gamma - (\alpha + \beta + 1)x]y' - \alpha\beta y = 0.$$

We find the representation of Kummer's twenty-four integrals in terms of hypergeometric series, provided the numbers γ, $\gamma-\alpha-\beta$, $\alpha-\beta$, are not integers; the representation of these integrals in form of curve integrals (following a general method of Euler); a complete discussion of the case in which the numbers mentioned before are integers; the study of the group of the hypergeometric equation, and also the determination of certain linear relations between various integrals.

The discussion of the logarithmic case is particularly valuable; it can not be found easily in the usual textbooks on differential equations.

In the formula (13) on p. 54 the term $(z^2-1)^n$ has to be replaced by $(1-z^2)^n$.

The present pamphlet is the first part of a monograph planned on hypergeometric series. Unfortunately the author died on November 26, 1936.

G. Szegö

Functions of Real Variables. By William Fogg Osgood. University Press, National University of Peking, 1936. 12+399 pp.

Functions of a Complex Variable. By William Fogg Osgood. University Press, National University of Peking, 1936. 8+257 pp.

These two texts are prepared for the student who has completed a course in "advanced calculus," as for example, one based upon the well known text by the author, and who is about to enter into the deeper mysteries of mathematical analysis. The second of the two works under review presupposes also some knowledge of functions of real variables but does not require as much as is handled in the first of these two volumes. One finds here well-organized courses, systematic, lucid, fundamental, with many brief sets of appropriate exercises, and occasional suggestions for more extensive reading. The technical terms have been kept to a minimum, and have been clearly explained. The aim has been to develop the student's power and to furnish him with a substantial body of classical theorems whose proofs illustrate the methods and whose results provide equipment for further progress. There is throughout a wholesome regard for steady application to essentials, with no vague references to diverting side issues. There is no room here for discussion of such special topics of increasing modern interest for students of real variables as summability of general divergent series, systems of orthogonal functions, or abstract spaces. Even the theory of Lebesgue measure is left for later study. So also in the briefer course in complex variables, Hadamard's three circles theorem and the discussion of the maximum modulus are of course not mentioned. Even the classical topic of elliptic functions claims less than ten pages. The lack of index to either volume seems an unnecessary handicap.

For the prospective teacher or student looking for a sound investment for his mathematical efforts the present works may be recommended. Perhaps the most pertinent information that this review could contain would be a brief outline of the contents of these respective volumes.

Functions of Real Variables. Chapter 1 deals with convergence of infinite series, including Kummer's criterion, a discussion of infinite products, and a very brief account of the hypergeometric series. Having thus whetted the reader's appetite for the study of limits, the author devotes Chapter 2 to the number system. Point sets, limits, and continuity constitute the subject matter of Chapter 3, which opens with fifteen examples of point sets, and proceeds to the covering theorem (usually called the Heine-Borel theorem), and the axiom of choice. The next two chapters give the classical theory of derivatives, integrals, implicit functions, and uniform convergence. Chapter 6 deals with the "elementary functions," trigonometric and exponential, discussing very briefly their important properties as functions of real variables, including mention of infinite product representations. In the next chapter, infinite series are studied systematically with regard to their algebraic manipulation. The previous chapters should serve to make the theory concretely applicable and further applications of interest are introduced, in particular the Legendre polynomials and the Bernoulli numbers. Chapter 8 gives a brief but substantial study of the basic ideas connected with Fourier series, even including Gibbs' phenomenon. The discussion of point sets, continuity, uniform convergence, Jordan curves, and other such topics given previously, is applied in Chapter 9 to the theory of definite integrals, including line integrals. Applications are made to classical integrands, "Duhamel's Theorem" is critically discussed, and the student is by this time fairly started in classical analysis. The final chapters are devoted to systematic study of the gamma function, the Fourier integral, and the existence theorems for solutions of differential equations.

Functions of a Complex Variable. The contents suggest in large part a simplification of the more elaborate treatise of the author on this same subject, printed in the German language. Chapters 1, 2, and 3 deal with the algebra and geometry of complex numbers and linear (fractional) transformations, with a study of the definition of analytic functions giving applications to some elementary functions. Riemann surfaces, illustrated by some very simple cases, constitute the subject of Chapter 4. The Cauchy theory leading to the Cauchy-Taylor development and disposing, in passing, of the "fundamental theorem of algebra," provides the material for Chapter 5. The further development of the theory of analytic functions as due chiefly to Weierstrass and Riemann, together with two more proofs of the "fundamental theorem of algebra," is given in Chapter 6. The chapter includes a treatment of residues, and further material on conformal mapping. Darboux's theorem is given and applied to the theory of the mapping of a rectangle on a circle, thus giving insight into an aspect of the elliptic integrals, although their name is nowhere mentioned in this chapter. Chapter 7 on "analytic continuation" discusses also the problem of uniformizing functions, in particular algebraic functions, and gives a survey of elliptic functions as approached through elliptic integrals. The last two chapters deal with the logarithmic potential and the conformal mapping of a simply connected region, with Picard's theorem a high point of the discussion.

The books are neatly printed in English and manifest the results of extraordinarily painstaking press work on the part of the Chinese publishers, and careful proofreading by the author. Save for the inevitable handicap of studying a subject in a foreign language, Chinese students may well be grateful for texts so clear, cogent, and fundamental.

A. A. Bennett