$$H^n(x) = \frac{R_n(x)}{(x-1)^n}.$$

The relation (7) gives many generalizations of (9). For example we can take $m=p^e$ in lieu of $m=p$. Further details I hope to give in another paper on Bernoulli numbers and Euler polynomials.

UNIVERSITY OF TEXAS

A THEOREM ON MEAN RULED SURFACES

BY MALCOLM FOSTER

Consider the ruled surface formed by the normals to a surface S along some curve C on S. We ask: What are the curves C for which the line of striction of the ruled surface is the locus of the centers of mean curvature corresponding to C?

On S we take the lines of curvature parametric. Referred to the moving trihedral of S, the direction-cosines of the normal are (0, 0, 1), and the variations in these are given by*

$$dX = qdu, \qquad dY = -p_1dv, \qquad dZ = 0.$$

Now the displacement of the central point on each generator of the ruled surface is orthogonal both to the normal and to its neighboring position. Hence we have

$$\delta z = 0, \qquad qdu\delta x - p_1dv\delta y + \delta z = 0,$$

which reduce to

$$qdu(\xi du + zqdu) - p_1dv(\eta_1dv - zp_1dv) = 0. \tag{1}$$

If in (1) we assign a value to the ratio dv/du, this equation will determine the distance z to the line of striction on the ruled surface defined by this ratio; and if to z we assign a given value, equation (1) will determine the curves, (though not necessarily real), for which this assigned value of z is the distance to the lines of striction.

From (1) we have for the problem at hand,

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* Eisenhart, *Differential Geometry of Curves and Surfaces*, pp. 166–174.

$$z = \frac{p_1\eta_1 dv^2 - q\xi du^2}{p_1^2 dv^2 + q^2 du^2} = \frac{q\eta_1 - p_1\xi}{2p_1 q},$$

which after some simplification may be written

$$(q\eta_1 + p_1\xi)(p_1^2 dv^2 - q^2 du^2) = 0. \tag{2}$$

It is readily seen that the vanishing of the first factor in (2) would mean that S is either a sphere or a plane. Hence, excluding these cases, the equation of the curves defining the mean ruled surfaces of the congruence of normals to S is given by*

$$q^2 du^2 - p_1^2 dv^2 = 0. \tag{3}$$

Since this is identical with the equation which defines the principal surfaces of the Ribaucour congruence for which S is the director surface, we have the following theorem.

THEOREM. *The mean ruled surfaces of the congruence of normals to a surface S are represented on S by curves which define also the principal surfaces of the Ribaucour congruence for which S is the director surface.*

WESLEYAN UNIVERSITY

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* Foster, *Rectilinear congruences referred to special surfaces*, Annals of Mathematics, (2), vol. 25 (1923), p. 177.