A CERTAIN THREE-DIMENSIONAL CONTINUUM*

BY W. T. REID

In this note there is given an example of a bounded continuum in three-dimensional euclidean space such that there exists a point A of M and subcontinua G, K_1, and K_2 of $M-A$ satisfying the following conditions: (1) $M-K_i$, $(i=1, 2)$, is the sum of two mutually separated point sets each of which is connected; (2) each point of G is separated from A in M by either K_1 or K_2; however, (3) there does not exist a point set consisting of a finite number of connected subsets of M and separating A from G in M.

This example was obtained in 1928 while the author was a student at the University of Texas. It answers in the negative for three-dimensional continua a certain question concerning continua proposed to the author by R. L. Moore at that time. R. E. Basye† has recently answered Moore's question in the affirmative for plane continua. It is because of Basye's result that the example of this note is now published.

EXAMPLE. Using rectangular coordinates, consider the points $p_i=(1/i, 0, 0)$, $p_0=(0, 0, 0)$, $t_i=(1/i, 0, 1)$, $t_0=(0, 0, 1)$, $s_i=(1/i, -1, 0)$, $s_0=(0, -1, 0)$, and $A=(0, 1, 0)$, $(i=1, 2, \cdots)$. If P and Q are two points we shall denote by PQ the closed straight line segment from P to Q. The continuum M is now defined as $M=\sum[Ap_n+p_ns_n+p_nt_n]+s_1s_0+s_0t_0+t_0t_1$, where it is understood that the summation is with respect to n over the values $n=0, 1, 2, \cdots$. The subcontinua G, K_1, and K_2 are defined as follows: $G=s_1s_0+s_0t_0+t_0t_1$, $K_1=\sum p_nt_n+t_0t_1$, $K_2=\sum p_ns_n+s_0s_1$. It is seen that $M-K_1=U_1+V$, where $U_1=\sum[s_np_n-p_n]+[s_0t_0-t_0]+s_0s_1$ and $V=\sum[Ap_n-p_n]$. Similarly, $M-K_2=U_2+V$, where $U_2=\sum[t_np_n-p_n]+[s_0t_0-s_0]+t_0t_1$. Clearly U_i and V, $(i=1, 2)$, are mutually separated point sets each of which is connected. Moreover, each point of G belongs to either U_1 or U_2. However, $M-(G+A)=\sum B_n$, where $B_n=\sum[Ap_n-A]+\sum[p_nt_n-t_n]+\sum[p_ns_n-s_n]$. Since for $n\neq m$, $(n, m=0, 1, \cdots)$,

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† See this issue of this Bulletin, pp. 670–674.

the connected point sets B_n and B_m have no point in common, there clearly exists no point set consisting of a finite number of connected subsets of M and separating G from A in M.

THE UNIVERSITY OF CHICAGO

A THEOREM ON PLANE CONTINUA*

BY W. T. REID

1. *Introduction.* In this paper the following theorem is proved.

THEOREM. *If M is a plane continuum, and K is a proper subcontinuum of M, then at least one component of $M-K$ has a limit point in K.*

Two points sets are *mutually separated* if they are mutually exclusive and neither of them contains a limit point of the other. A point set is said to be *connected* if it is not the sum of two non-vacuous mutually separated point sets. A point set which is both connected and closed is a *continuum*. A *component* of a point set N is a connected subset of N which is not a proper subset of any other connected subset of N. The set of all points in the plane will be denoted by S.†

2. *Proof of the Theorem.* If M is a bounded continuum and K is a proper subcontinuum of M, it is well known that every component of $M-K$ has a limit point in K.‡ If M is unbounded then it is no longer true that *every* component of $M-K$ has a limit point in K.§

If K is a bounded subcontinuum of an unbounded plane continuum M, then the above theorem may be proved readily. For

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* Presented to the Society, December 28, 1934. The result of this paper was obtained in 1928, while the author was a student under R. L. Moore at the University of Texas. Recently both R. L. Moore and J. H. Roberts have proved results beyond that of the present paper and have suggested that I publish my original result.

† These definitions are those customarily used in point set theory. See, for example, R. L. Moore, *Foundations of Point Set Theory*, Colloquium Publications of this Society, vol. 13. For brevity, this treatise will be referred to as "Moore."

‡ See, for example, Moore, p. 24.

§ See Moore, p. 25, example 2.