(c) $A_2 = 8\sum q^n(\sum (-1)^d \delta \cos 2dy) - 8\sum q^n(\sum \delta)$,

(d) $A_3 = 1 + 8\sum q^n(\sum (-1)^t \tau \cos 2ty) - 8\sum q^n(\sum \delta)$,

(e) $B_0 = 1/2 + 2\sum q^n(\sum \delta) + 8\sum q^n(\sum (2\tau - t) \cos 2ty)$,

(f) $B_1 = -3/2 + \csc^2 y + 2\sum q^n(\sum \delta)$
$$+ 8\sum q^n(\sum (2\delta - d) \cos 2dy),$$

(g) $B_2 = -3/2 + \sec^2 y + 2\sum q^n(\sum \delta)$
$$+ 8\sum q^n(\sum (2\delta - d)(-1)^d \cos 2dy),$$

(h) $B_3 = 1/2 + 2\sum q^n(\sum \delta)$
$$+ 8\sum q^n(\sum (-1)^t(2\tau - t) \cos 2ty).$$

University of Arkansas

A CONNECTEDNESS THEOREM IN ABSTRACT SETS*

BY W. M. WHYBURN

This note gives a variation of a theorem of Sierpinski and Saks.† The theorem is valid in spaces which have the Borel-Lebesgue property (Axiom I of Saks‡) and which satisfy axioms (A), (B), (C), and (6) as given by Hausdorff.§ We use the term *connected* for a closed set to mean that the set cannot be expressed as the sum of two mutually exclusive non-vacuous, closed sets.||

Theorem. *Let F be a collection of closed sets at least one of which is compact. Let F contain more than one element and let it be true that the sets of each finite sub-collection of F have a non-vacuous, connected set in common when this sub-collection contains at least two elements of F. Under these hypotheses, there is a closed, non-vacuous, connected set common to all of the sets of collection F.*

Proof. Let F_0 be a compact member of collection F and let K be the set of points common to all of the sets of collection F.

---

* Presented to the Society, December 1, 1934.

† See Saks, Fundamenta Mathematicae, vol. 2 (1921), pp. 1–3.

‡ Saks, ibid., p. 2.

§ *Mengenlehre*, 1927, pp. 228–229.

|| The notion of *limit point* may be defined and this definition used to describe connectedness. We use *domain* and *open set* interchangeably.

Saks* shows that K is non-vacuous while the closure of K is an immediate consequence of the closure of the sets of collection F (since any point of the complement of K has a neighborhood which belongs to the complement of some one of the sets of collection F and hence belongs to the complement of K). It remains to show that K is connected. Suppose $K=K_1+K_2$, where K_1 and K_2 are mutually exclusive, non-vacuous, closed sets. The collection C composed of the complements of the sets of collection F is a set of domains that covers F_0-K. By axiom (6), for each point p of K_1 there exist mutually exclusive domains G_{1p} and G_{2p} such that $p \epsilon G_{1p}$, $K_2 \subseteqq G_{2p}$. Let $[G_{1p}]$ and $[G_{2p}]$ be the collections of domains obtained in this manner for all points of K_1. The set K_1 is closed and compact and hence has the Borel-Lebesgue property. Let $G_1, \cdots, G_n$ be a finite sub-collection of $[G_{1p}]$ which covers K_1 and let $H_1, \cdots, H_n$ be the corresponding members of $[G_{2p}]$. If H denotes the common part of $H_1, \cdots, H_n$, then H is a domain that covers K_2 (this follows from a theorem stated by Hausdorff, loc. cit., page 229, line 4) while $G=G_1+G_2+\cdots+G_n$ is a domain that covers K_1. Furthermore, H and G have no point in common since G_i and H_i are mutually exclusive sets. The collection C together with G and H cover the closed and compact set F_0. The Borel-Lebesgue property yields a finite collection $C_1, C_2, \cdots, C_m, G, H$, of these sets that covers F_0 while the hypotheses of the theorem together with the method of construction of the covering sets force this collection to contain G, H, and at least one of the sets C_i. Let F_i be the complement of C_i and let Q be the set common to $F_0, F_1, \cdots, F_m$. The set Q contains K_1 and K_2 and is covered by G and H (since Q belongs to F_0 and the complements of $C_1, C_2, \cdots, C_m$). Since G and H are mutually exclusive, it follows that Q is not connected. This contradicts the hypothesis that any finite collection of two or more of the sets of F has a connected set in common and yields the theorem.

The University of California at Los Angeles

---

* Loc. cit., p. 2.