SHORTER NOTICES

Les Surfaces Algébriques non Rationnelles de Genre Arithmétique et Géométrique Nuls. By L. Godeaux. (Actualités Scientifiques et Industrielles, No. 123.) Paris, Hermann, 1934. 33 pp.

The most important invariant of an algebraic curve is its genus. A necessary and sufficient condition that a non-composite curve shall be rational is that its genus be zero. An algebraic surface has an infinite number of genera, each a direct and natural analog of the genus of a curve. These are the arithmetic genus p_a, the geometric genus p_g, and the series of multigenera P_i. It was shown by Castelnuovo, immediately after Enriques had discovered the invariants P_i, that a necessary and sufficient condition for rationality is $p_a=0$, $P_2=0$. Later, each of these scholars actually constructed irrational surfaces having $p_a=0$, $p_g=0$, $P_2\neq 0$.

The present pamphlet clearly states the problem, shows that these illustrations are birationally distinct, gives the further case found by Campedelli, $P_2=2$, and adds a new one for which $P_2=2$, $P_3=4$. The method of proof is entirely algebraic, using only the well known theorems concerning grade, genus, and laws of combination for linear systems of curves on an algebraic surface. An excellent introduction to this memoir is the paper by Roberta F. Johnson, *Involutions of order two associated with the surfaces of genera* $p_a=p_g=0$, $P_2=1$, $P_3=0$ (American Journal of Mathematics, vol. 56 (1934), pp. 199–213).

VIRGIL SNYDER

Mathematical Problems of Radiative Equilibrium. By Eberhard Hopf. Cambridge Tracts in Mathematics and Mathematical Physics. New York, Macmillan. viii+105 pp.

This is an excellent account of the mathematical developments associated with a class of integral equations in which the kernel is a particular type of function of the modulus of the difference of x and y. The author regards his work as applicable to the mathematical problems of radiative equilibrium which arose from the work of Schuster and Schwarzschild on a purely scattering atmosphere; and became more intricate when the atmosphere was regarded as absorbing and emitting radiation as in the work of Schwarzschild, Humphreys, Gold, and Emden on the earth's atmosphere and in the later work of Eddington, Jeans, Milne, and others on stellar atmospheres and planetary nebulae. It may be suggested, however, that the analysis may also be useful for a continuation of the work of E. Lommel, *Die Photometrie der diffusen Zurückwerfung*, published in 1889 in volume 36 of Wiedemann's Annalen der Physik und Chemie, for in this work Lommel obtains an integral equation of the type under consideration, the particular function being the exponential integral which belongs to the type mentioned in this tract. It is impossible to give a brief summary of the analysis, but it can be said that the work is of a high standard throughout, much use being made of inequalities and convergence theorems. It is a useful contribution to modern astrophysics.

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