problem of the transverse vibration of a string and the classical boundary value problem of harmonic functions (the Dirichlet and Neumann distributions) are treated. This seems much too brief even for an elementary exposition of the theory of integral equations. It also seems rather unfortunate that almost no references to literature are made in the text. The author refers to the article of Hellinger and Toeplitz in the Encyklopädie der Mathematischen Wissenschaften as supplying completely the need for bibliographical material. In a book designed particularly for instruction, it seems unfortunate that more reference to the historical development and to the principal contributions has been omitted. The book could have been easily illuminated by such reference.

H. T. DAVIS

The Theory of Matrices. By C. C. MacDuffee. (Ergebnisse der Mathematik und ihrer Grenzgebiete.) Berlin, Julius Springer, 1933. 110 pp.

To review in any really critical way such a book as this is impossible. The editors of the "Ergebnisse" series carefully pick one of the three or four men in the world that know most about matrices. That man spends a year or two of intensive reading of the literature, of intensive thought on both the details and organization of the subject, and then a reviewer is supposed to act learned and critical. It just isn't a proper set-up.

It is possible to point out what the author attempts to do and what he does not attempt and to give some idea as to how useful is this particular piece of work, under the amply fulfilled hypothesis that the workmanship is of high order.

Much adverse criticism, sometimes written but more often spoken, of scientific publications is based, not on how well a piece of work is accomplished, but on the decision of the reviewer as to whether he would like the author to have written something else instead. This is unfair. The reviewer's feeling is that we clearly need original contributions of high order, we clearly need organizing works which unify bodies of seemingly diverse doctrines, and we clearly need encyclopaedic discussions that make available a large body of already written material, and that we should welcome any book that fulfills any one of these purposes. This work definitely is of the encyclopaedic type though, owing to the author's search for elegant proofs and to the necessity of making each theorem depend on preceding work, there has been brought about a considerable amount of unity. Though this book is not lacking in original material, the author's personal contributions play a minor part and have been mostly published elsewhere.

The outline of the book is told simply by the chapter headings, which are as follows: 1. Matrices, arrays and determinants. 2. The characteristic equation. 3. Associated integral matrices. 4. Equivalence. 5. Congruences. 6. Similarity. 7. Composition of matrices (this chapter including questions dealing with direct sum and products, etc.). 8. Matric equations. 9. Functions of matrices. 10. Matrices of infinite order. Except for the fact that the author has a strong preference for the theory of linear algebras and especially for its associated number theory, this is just what anyone interested in matrix theory would use as an outline. The great value of the book, and its value is great, lies in the fact that a large amount of detail is presented in outline, with well arranged refer-

ences to the literature, and in such a way that one can understand the relations of the work of various authors to the field as a whole and become acquainted with this work with a minimum of labor. Much of this material is nowhere else available except in the original memoirs, and thus for the first time really takes its place in the body of mathematics. The reviewer has not the knowledge to give final judgment as to how completely the field is gleaned, but although he is fairly familiar with certain portions of matrix theory, there was no portion in which the reading of this book did not only bring to his attention references of which he had no previous knowledge but facts that he did not know.

The most striking deliberate omission is the result of a clear cut policy of not describing the applications of matrix theory, although, of course, a knowledge of these applications is responsible for the inclusion of much material.

The reviewer would not be human if there were no items of irritation in such a work. He doesn't like, for instance, the use A^I for the reciprocal of A, but he doesn't like spinach either, and both are matters of personal taste. He doesn't like the postulational definition of equality because he likes an introduction of the equality symbol by another means far nearer the logical foundations of mathematics; again this is a matter of taste. The fundamental thing is that the book is a high grade piece of valuable work and the reviewer's grievances, though forming basis for delightful evenings of argument, are petty and have no essential part in a review.

No mathematical library can afford to be without this book. Every worker in matrix theory will find it both enriching to his knowledge and a great time-saver. With all this the author has had the saving grace in each portion to make the reader feel how much is left to be done both by way of solving extremely difficult problems and by way of rather routine and methodical progress. In fact, the reviewer shudders to think how many doctors' dissertations will be the result of this definition of the boundary of knowledge in this one field.

The author has asked that the following corrections be noted:

1. Page 31, footnote 1: This follows from van der Waerden's definition of principal ideal ring, but not from the definition given on p. 29.
2. Page 43, line 23: $h_i = p_1 e^{i\iota_1}$, etc. line 26: $p_l e^{i\iota_l}$.
3. Page 82, footnote 1: Add Zehfuss, G.: Zeitschrift für Mathematik und Physik, vol. 3 (1858), p. 298.
4. Page 93, lines 3 and 12: Replace m_n by m_{n-1}.
5. Page 93: Delete three lines beginning "A. Voss."[12]
6. Page 101, line 1: Replace "small closed path" by "closed path inclosing all the characteristic roots of A."
7. In Chapter X mention should be made of the work of S. Pincherle on *Linear Operations*. For a list of his writings, see Acta Mathematica, vol. 46 (1925), p. 357.

The reviewer also wishes to correct another error. The theorem 36.5 on page 63 is not true, the condition given being necessary but not sufficient. In a recent abstract Mr. K. W. Wegner gives the complete facts for the non-singular case. The singular case is now under consideration.

M. H. INGRAHAM