Integralgleichungen. By G. Kowalewski. Berlin and Leipzig, de Gruyter, 1930. 11 figures. 302 pp.

To those who have derived inspiration and guidance from the lucid account given in Kowalewski's *Einführung in die Determinantentheorie* (1909) of the theory of infinite determinants and its application to integral equations, the present volume comes as a welcome addition to the literature of this subject. The book was obviously written as a text and not as a treatise, the author defining his purpose in the preface as follows: "Several chapters in my book on determinants appearing in 1909 were the first textbook presentation of the theory of integral equations in Germany. In the second edition, since sharp abbreviation was necessary, a considerable part of these chapters was omitted. I was again invited to write a special book on integral equations. Thus the present work was commenced. It is to be regarded as a book for our students."

The book consists of an extensive introduction and four chapters headed as follows: Integral Equations of Volterra Type; Integral Equations of Fredholm Type; Fredholm Equations with Symmetric Kernel; Several Applications of Integral Equations.

The book is noteworthy for its clear exposition of the subject. It does not make extensive excursions into recent developments, the discussion of singular equations, etc., but adheres closely to what might be called the classical theory. The book opens with adequate discussions of the Abel equation and the Fourier transform in its integral equation form. The connection between differential and integral equations is discussed more thoroughly than in most texts. The Cauchy problem, in which conditions are imposed at one point only, is reduced to the solution of a Volterra integral equation; the Lagrange problem, where conditions are imposed at more than one point, is shown to lead through Green's functions to Fredholm integral equations.

The Volterra integral equation is treated by standard methods and no attempt is made to go into the case of singular kernels except for the elementary case $k(x, y)/(x-y)^m$, $(0 \leqq m < 1)$.

Most of the book is devoted to the Fredholm equation. The author has done an excellent thing from the standpoint of exposition by basing much of his argument and discussion on the bilinear kernel, $K(x, y) = \sum_{i=1}^{n} A_i(x)B_i(y)$. It will be recalled that the original derivation of the Fredholm theory from this point of view was made by E. Goursat (Bulletin de la Société Mathématique de France, vol. 35 (1907), pp. 163–173) and H. Lebesgue (ibid., vol. 36 (1908), pp. 3–19), the former developing the formal theory and the latter extending Goursat's results to the infinite case. This approach, rather than the one through the limiting form of a set of algebraic equations, has much to recommend it as an introduction to the theory of the Fredholm equation. Actual solutions can be attained for special cases and properties of the Fredholm determinant can be developed as properties of polynomials. The author makes admirable use of the Goursat-Lebesgue kernel to display many of the attractive theorems associated with the theory of elementary divisors without complicating the picture by means of infinite determinants and matrices. It must be added, however, that these modern tools are used freely throughout the book.

The book concludes with a very brief chapter on applications. Only the problem of the transverse vibration of a string and the classical boundary value problem of harmonic functions (the Dirichlet and Neumann distributions) are treated. This seems much too brief even for an elementary exposition of the theory of integral equations. It also seems rather unfortunate that almost no references to literature are made in the text. The author refers to the article of Hellinger and Toeplitz in the Encyklopädie der Mathematischen Wissenschaften as supplying completely the need for bibliographical material. In a book designed particularly for instruction, it seems unfortunate that more reference to the historical development and to the principal contributions has been omitted. The book could have been easily illuminated by such reference.

H. T. Davis

The Theory of Matrices. By C. C. MacDuffee. (Ergebnisse der Mathematik und ihrer Grenzgebiete.) Berlin, Julius Springer, 1933. 110 pp.

To review in any really critical way such a book as this is impossible. The editors of the "Ergebnisse" series carefully pick one of the three or four men in the world that know most about matrices. That man spends a year or two of intensive reading of the literature, of intensive thought on both the details and organization of the subject, and then a reviewer is supposed to act learned and critical. It just isn't a proper set-up.

It is possible to point out what the author attempts to do and what he does not attempt and to give some idea as to how useful is this particular piece of work, under the amply fulfilled hypothesis that the workmanship is of high order.

Much adverse criticism, sometimes written but more often spoken, of scientific publications is based, not on how well a piece of work is accomplished, but on the decision of the reviewer as to whether he would like the author to have written something else instead. This is unfair. The reviewer's feeling is that we clearly need original contributions of high order, we clearly need organizing works which unify bodies of seemingly diverse doctrines, and we clearly need encyclopaedic discussions that make available a large body of already written material, and that we should welcome any book that fulfills any one of these purposes. This work definitely is of the encyclopaedic type though, owing to the author's search for elegant proofs and to the necessity of making each theorem depend on preceding work, there has been brought about a considerable amount of unity. Though this book is not lacking in original material, the author's personal contributions play a minor part and have been mostly published elsewhere.

The outline of the book is told simply by the chapter headings, which are as follows: 1. Matrices, arrays and determinants. 2. The characteristic equation. 3. Associated integral matrices. 4. Equivalence. 5. Congruences. 6. Similarity. 7. Composition of matrices (this chapter including questions dealing with direct sum and products, etc.). 8. Matric equations. 9. Functions of matrices. 10. Matrices of infinite order. Except for the fact that the author has a strong preference for the theory of linear algebras and especially for its associated number theory, this is just what anyone interested in matrix theory would use as an outline. The great value of the book, and its value is great, lies in the fact that a large amount of detail is presented in outline, with well arranged refer-